

MAXIMAZATION OF PROFIT FROM SOAP PRODUCTION USING LINEAR PROGRAMMING OPTIMIZATION TOOL**Nitonye, Boma, Dr. Isaac, O.E., Dr. Nkoi, B. and *Prof. Abomaye-Nimenibo, Williams Aminadokiari Samuel**

Department of Mechanical Engineering, Faculty of Engineering, Rivers State University, Nkpolu-oroworukwo, Port Harcourt, and Obong University/Charisma University

Received 20th November 2022; Accepted 27th December 2022; Published online 25th January 2023

Abstract

This research sought to perform profit maximization for soap production using linear programming. The decision variables in this research were the five different types of soap (Bar soap, Aura toilet soap, Medicare Medicated soap, Translucent soap and Jumbo-all-purpose soap) produced by Rivers State Vegetable Oil Coy (RIVOC). Seven raw materials (sodium hydroxide or caustic soda, sodium silicate, perfumes, dye or colour, Palm kernel oil, salt and production time) were used in the soap production and the current quantities of materials for maximum profit in soap production as well as the time taken to produce each of the different variables of soap was determined. Linear programming optimization tool based on the soap production variables and constraints with the objective function of maximizing profit for the soap industry was applied, and the product mix and schedule of raw materials in soap production for maximum profit was determined using linear programming with LINDO software. Results showed that for profit maximization, 1052.63 units of Jumbo-all-purpose soap only should be produced daily amongst the other soap types, and with this product mix, a maximum profit of ₦68, 421.05 was generated daily. The result showed that producing 1052.63 units of Jumbo-all-purpose soap and zero production of the other types of soap (Bar soap, Aura toilet soap, Medicare Medicated soap, Translucent soap) generated a maximum profit of ₦68, 421.05 daily. Conclusion and recommendations were made that RIVOC Soap Production Industry should produce five different types of soap, but more Jumbo-all-purpose soap should be produced to attain maximum profit.

Keywords: Profit Maximization, Soap Production, Raw Materials, Linear Programming Optimization Tool, and LINDO Software.

1. INTRODUCTION**1.1 Background of the Study**

Production is one of the key functions in organizations, and it is concerned with the transformation of input resources into required outputs (products). Production involves the conversion of one form of material into another form through a chemical or mechanical process to create or enhance the utility of the product to the users (Buffa, 2001). Manufacturing industries are usually faced with limited scarce resources including people, machines, time and materials. Proper allocation and maximization of these resources enhance organizational efficiency and effectiveness in meeting goals and making a profit. It is a value-addition process. Poor resource maximization may endanger the financial health and survival of organizations (Nsikan & Okeh, 2017). Hence, for an organization to maximize profit, the production of raw materials and scarce resources must be optimally utilized (Banjoko, 2002). Production optimization deals with maximizing the scheduling of jobs, allocation of scarce resources, assigning workloads to machines and people as well as the actual flow of work (Amole *et al.*, 2016). Production optimization estimates the resources required and prepares a detailed plan for achieving the production goals efficiently and timely (Amole *et al.*, 2016). Production optimization is a basic function of manufacturing management applicable in all manufacturing companies in which it shows the direction and coordination of firms' resources towards attaining their prefixed goals (Kumar & Suresh, 2008). However, the production itself is an organized activity that leads to the transformation of raw materials into useful products.

For a successful production, a series of activities are involved. These include effective maximization of natural resources such as workers, finance, equipment, materials and time. Production optimization is associated with maximizing the usage and available quantity of the resources required to perform transformation and manufacturing processes, to satisfy the customers most efficiently or economically. Production optimization is the direction and coordination of a firm's resources towards attaining the prefixed goals. It helps to achieve an uninterrupted flow of materials at the right time and required quality. Production is regarded as the centre focus of any manufacturing organisation concerned. It is a stage where there is a transformation of raw materials (inputs) into finished products (outputs). Production is a process or procedure developed to transform a set of inputs like men, materials, capital, information and energy into a specified set of outputs like finished products and services in proper quantity and quality, thus achieving the objectives of an enterprise (Vollman *et al.*, 2007; Jain and Aggarwal, 2008). The production system likewise is the design process by which elements are transformed into useful products. The designed process is an organized procedure for accomplishing the conversion of inputs into output. The optimization of operations is necessary because in manufacturing units there are usually limited scarce resources including people, machines and materials. Proper allocation and usage of these resources enhance organizational efficiency and effectiveness in meeting goals and making a profit. Indeed, there will be an evident improvement in the economy if sectors maximize productivity with few resources as Akpan and Iwok (2016) opined that an economy can only grow if management decisions at the firm level result in boosted output through cost minimization or output maximization culminating in increased production in the real sector. Failure to minimize cost will

make it more difficult for a manufacturing organization to maximize profit or benefit. However, to minimize cost, it is pertinent for a production manager to decide on the best way to allocate limited resources in such a manner that it will lead to higher output and profit. It is for this reason that the research has decided to apply linear programming as a production optimization technique to improve the profitability of the soap production industry in Nigeria. Recent works of literature show that a lot of attention has been paid to the problem of production optimization in manufacturing systems and industries. Many authors are professionally engaged in Karmarkar's Approach for Solving Linear Programming Problems for Profit Maximization in Production Industries: NBC Port-Harcourt Plant, and profit optimization using simplex methods on home industry Bintang bakery in Sukarame Bandar Lampung (Nsikan & Okeh, 2017; Bambang *et al.*, 2019). Sulaimon *et al.* (2014) studied linear optimization techniques for the product mix of paint production in Nigeria. Benedict and Uzochukwu (2012) presented an approach to optimizing profit with the linear programming model: A focus on golden plastic industry limited, Enugu, Nigeria.

Recent works of literature show that a lot of attention has been paid to the problem of production optimization in manufacturing systems and industries. Many authors are professionally engaged in Karmarkar's Approach for Solving Linear Programming Problems for Profit Maximization in Production Industries: NBC Port-Harcourt Plant, and profit optimization using simplex methods on home industry Bintang bakery in Sukarame Bandar Lampung (Nsikan & Okeh, 2017; Bambang *et al.*, 2019). Sulaimon *et al.* (2014) studied linear optimization techniques for the product mix of paint production in Nigeria. Benedict and Uzochukwu (2012) presented an approach to optimizing profit with the linear programming model: A focus on golden plastic industry limited, Enugu, Nigeria.

Okolie *et al.* (2014) studied the optimization of a soap production mix using response surface modelling: A Case of Niger Bar Soap Manufacturing Industry Onitsha, Anambra State, Nigeria. Their study improved the productivity of Soap mix using Response Surface and presented the optimization of a soap production mix using the previous production data. The data were optimized using a response surface modelling tool to observe the optimum production mix of the raw material. Response surface regression analysis was used to estimate the coefficients for Y using data in coded units where the coefficient of determination (R-sq) is 100%. The response surface optimization model shows that the optimum production mix quantity of the soap should be 364.999kg.

Balogun, *et al.* (2013) studied an optimization procedure in a production line of Sokat Soap Industry, Ikotun, Lagos State. Their research work was used to demonstrate the application of linear programming in optimizing the profit in a production line using SOKAT Soap Industry, Ikotun, Lagos State. Available to the authors were the units of profit of all the products as well as the stocks available for the production line. Using these, the objective function was formulated subject to several constraints based on their practices. Optimization (maximization) was obtained by employing a linear programming technique. It was observed from the analysis of the data that the product which contributed mostly to the profit earned was the 1kg Salem soap. However, the establishment needed to produce others because of the need of their

numerous customers (the hotels and other institutions) that needed the other smaller sizes such as the 10kg, 5kg and 2kg soap. Linear programming is a family of mathematical programming that is concerned with production optimization or is useful for the allocation of scarce or limited resources to several competing activities based on a given criterion of optimality (Sulaimon *et al.*, 2014). In statistics, linear programming (LP) is a special technique employed in operation research for the optimization of a linear function subject to linear equality and inequality constraint. Linear programming determines the way to achieve the best outcome, such as maximum profit or minimum cost in a given mathematical model and given some list of requirements as a linear equation (Aadharshana, 2019). The technique of linear programming can be used for production optimization in a wide range of applications including agriculture, industry, transportation, economics, health system, social science and the military (Harvey, 2020; Bambang *et al.*, 2019). To facilitate and accelerated the completion of the calculation of linear programming mathematical model for production optimization is assisted by using a computer program (software) application. There is quite some software for linear programming simulations such as LiPs, GAMS, CPLEX, MOSEK, TORA, QSB, and LINDO to mention but a few (Sharma, 2009). However, this research will employ the LINDO software for production optimization in the soap production industry. Linear Interactive and Discrete Optimizer (LINDO) is a computer software package for linear programming, integer programming, nonlinear programming, stochastic programming and global programming (LINDO Systems Inc., 2003). In this research work, the raw materials and other production factors for the manufacturing of soap in Rivers State Vegetable Oil Company (RIVOC) were identified, assessed and understudied and this study utilized linear programming with the LINDO computer tool for optimization of soap production raw materials in RIVOC soap industry in Nigeria, to determine the optimal product mix and schedule of raw material allotment in the soap industry for profit maximization.

1.2 Statement of the Problem

The rate at which production companies liquidate in Nigeria is alarming. This can be traced to improper resource allocation, wastage of production resources, high level of idleness on the production floor, and the inability of the production companies to maximize the meaningful profit that will enable these companies to meet up with the expectations of their stakeholders (Maravelias and Sung, 2008; Ovunda *et al.*, 2019). Although several studies have dealt with production optimization in manufacturing industries, there hasn't been any research on profit maximization for soap production using LINDO optimization software. The consequences of poor production resources maximization in the soap manufacturing process lead to wastage of production resources, overtime, cost shooting up, high level of idleness on the production floor, production losses, unit cost rising as well as dissatisfaction among customers the major concern for this research work as it affects the manufacturing process in terms of cost and time. Therefore, performing profit maximization for soap production using LINDO optimization software is the motivation of this research work. Production managers must keep idle time on the production floor to a minimum, excess idle time tends to waste resources on the production floor. There is also a need to identify products that contribute maximally to profit and those

that yield less profit. This study is set to utilize the LINDO linear programming computer tool to optimize the production of raw material resources for the soap industry in Nigeria to realize an optimal profit and to maximize the use of their production resource efficiently.

1.3 Objective of the Study

This research work was carried out to ascertain that maximizing profit in soap production using linear programming in Rivers State Vegetable Oil Company is conceivable.

The specific objectives of the research are:

- i. To investigate the current quantity of materials for maximum profit in soap production.
- ii. To understudy the time taken to produce the different variables of soap.
- iii. To determine the product mix and schedule of raw materials in soap production for maximum profit using linear programming.

1.4 Significance of the Study

The significances of this study are as follows:

- i. The study will assist manufacturing industries to optimally allot production resources for the variables of their production system that will produce the most profit after sales.
- ii. The study will help in resource allocation to enhance increased output, increased competitiveness, increased profit and customer satisfaction.

2. MATERIALS AND METHOD APPLICATIONS

2.1 Materials

The materials used for this study were obtained from Rivers State Vegetable Oil Company (RIVOC), Trans-Amadi Industrial Layout, Rivers State, Nigeria. Details are stated below.

2.1.1 Raw Materials for Soap Production

Soap production in RIVOC makes use of these constituent raw materials: palm kernel oil, dye, resin, perfumes, magnesium sulphate, sodium hydroxide (caustic soda) and sodium silicate as presented in Figure 2.1(RIVOC, 2006b).

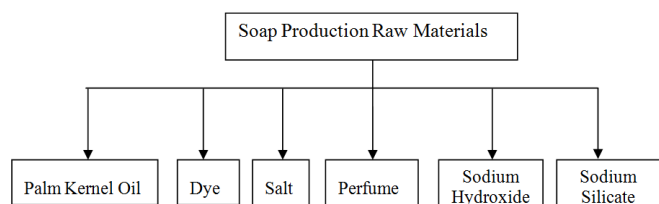


Figure 2.1. Raw Materials for Soap Production (RIVOC, 2006b)

Linear Interactive and Discrete Optimizer (LINDO) program is an algorithm, that is, a mathematical set of programs (mathematical software). It is an optimization system in the area of operations research. The Linear Interactive and

Discrete Optimizer (LINDO) computer program provides linear programming solutions that will be useful and available to the management of production industries in reducing wastages of their production system resources and set-up times which consequently lead to reduced expenses (LINDO Systems Inc., 2003). The study will also assist manufacturing industries to determine an optimal product mix for their production process flow line that will produce the most profit after sales. The quantities of the raw materials (sodium hydroxide or caustic soda, sodium silicate, perfumes, dye or colour, Palm kernel oil and salt) content per each unit product of soap (bar soap, aura soap, medicated soap, Translucent soap and Jumbo-all-purpose soap) manufactured in RIVOC are presented as follows:

i. Palm Kernel Oil /Other Fatty Matters

The total amount of Palm Kernel Oil /other Fatty Matter available = 1500kl

- Each unit of Bar soap requires 0.36kl of Palm Kernel Oil
- Each unit of Aura toilet soap requires 0.20kl of Palm Kernel Oil
- Each unit of Medicated soap requires 0.25kl of Palm Kernel Oil
- Each unit of Translucent soap requires 0.20kl of Palm Kernel Oil
- Each unit of Jumbo-all-purpose soap requires 0.30kl of Palm Kernel Oil

ii. Dye or Colour

- The total amount of Dye or colour available = 2000kg
- Each unit of Bar soap requires 0.025kg of Dye or colour
- Each unit of Aura toilet soap requires 0.020kg of Dye or colour
- Each unit of Medicated soap requires 0.015kg of Dye or colour
- Each unit of Translucent soap requires 0.015kg of PDye or colour
- Each unit of Jumbo-all-purpose soap requires 0.025kg of Dye or colour

iii. Salt

- Total amount of Salt available = 1000kg
- Each unit of Bar soap requires 0.012kg of Salt
- Each unit of Aura toilet soap requires 0.008kg of Salt
- Each unit of Medicated soap requires 0.005kg of Salt
- Each unit of Translucent soap requires 0.010kg of Salt
- Each unit of Jumbo-all-purpose soap requires 0.015kg of Salt

iv. Perfumes

- Total amount of Perfumes available = 500kg
- Each unit of Bar soap requires 0.0005kg of Perfume
- Each unit of Aura toilet soap requires 0.004kg of Perfume
- Each unit of Medicated soap requires 0.0007kg of Perfume
- Each unit of Translucent soap requires 0.0009kg of Perfume
- Each unit of Jumbo-all-purpose soap requires 0.001kg of Perfumes

v. Sodium Hydroxide/Caustic Soda

- Total amount of caustic soda available = 1200kg
- Each unit of Bar soap requires 0.22kg of caustic soda
- Each unit of Aura toilet soap requires 0.18kg of caustic soda

Each unit of Medicated soap requires 0.15kg of caustic soda
 Each unit of Translucent soap requires 0.15kg of caustic soda
 Each unit of Jumbo-all-purpose soap requires 0.20kg of caustic soda

vi. Sodium Silicate

Total amount of Sodium silicate available = 300kg
 Each unit of Bar soap requires 0.15kg of Sodium silicate
 Each unit of Aura toilet soap requires 0.06kg of Sodium silicate
 Each unit of Medicated soap requires 0.10kg of Sodium silicate
 Each unit of Translucent soap requires 0.03kg of Sodium silicate
 Each unit of Jumbo-all-purpose soap requires 0.15kg of Sodium silicate

2.1.2 Saponification- Soap-Making Processes

Saponification is defined as a process that produces soap, usually from fats and lye. In technical terms, saponification involves base (usually caustic soda NaOH) hydrolysis of triglycerides, which are esters of fatty acids, to form the sodium salt of a carboxylate. In addition to soap, such traditional saponification processes produce glycerol. "Saponifiable substances" are those that can be converted into soap (Okolie *et al.*, 2014). Knowledge of saponification is relevant to many technologies and many aspects of everyday life. Depending on the nature of the alkali used in their production, soaps have distinct properties. Sodium hydroxide (NaOH) gives "hard soap", whereas, when potassium hydroxide (KOH) is used, a soft soap is formed (Okolie *et al.*, 2014). The industrial production of soap involves continuous processes, such as the continuous addition of fat and the removal of products. Smaller-scale production involves the traditional batch processes. The three variations are: the 'cold process', wherein the reaction takes place substantially at room temperature, the 'semi-boiled' or 'hot process', wherein the reaction takes place near the boiling point, and the 'fully-boiled process', wherein the reactants are boiled at least once and the glycerol is recovered. There are two types of 'semi-boiled' hot process methods. The first is the ITMHP (in the mould hot process) and the second is the CPHP (Crockpot hot process). Typically soap makers choose the hot process method if they wish to remove the cure time to a three-day air-dry process. Most soap makers, however, continue to prefer the cold process method. The cold process and hot process (semi-boiled) are the simplest and typically used by small artisans and hobbyists producing handmade decorative soaps. The glycerine remains in the soap and the reaction continues for many days after the soap is poured into moulds. The glycerine is left during the hot-process method, but at the high temperature employed, the reaction is practically completed in the kettle, before the soap is poured into moulds. This simple and quick process is employed in small factories all over the world (Okolie *et al.*, 2014). Figure 3.2 illustrates the sequential steps involved in the use of these two processes for soap production.

The data obtained from this company was analysed using linear programming (LP), to maximize profit through optimal production of these items. The linear programming model developed was simulated and analysed using the LINDO

mathematical software. It is an optimization software in the area of operations research.

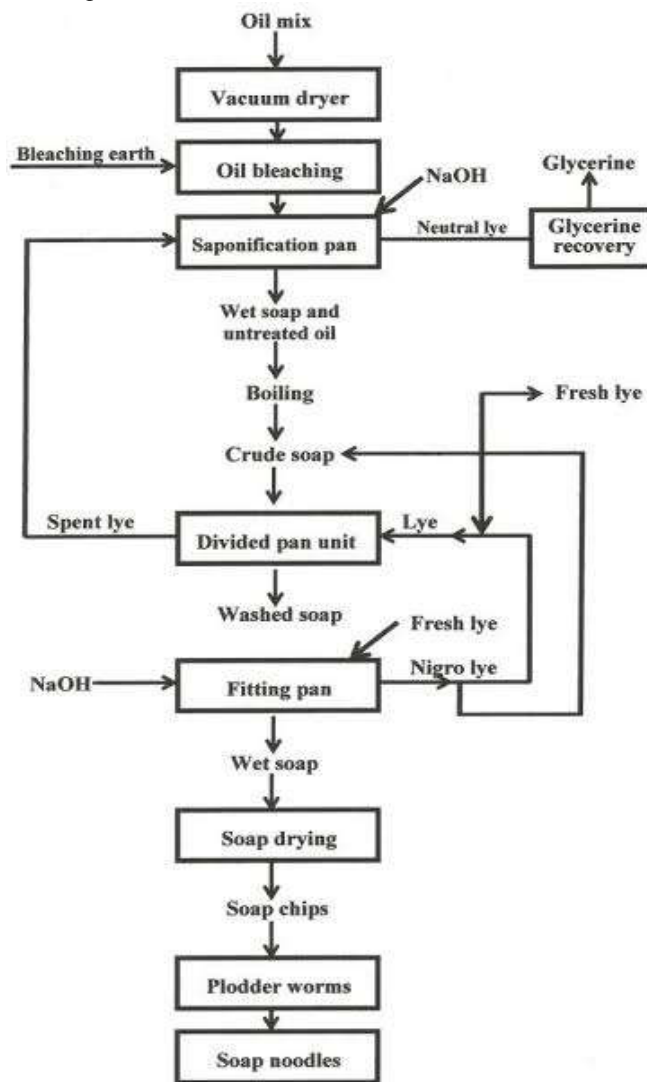


Figure 2.2. Flow Diagram of Soap Production by Batch Process (Okolie *et al.*, 2014)

2.1.3 Data Collection

The data for this research was obtained from production information of the products made available by the management of Rivers State Vegetable Oil Company (RIVOC), Trans-Amadi Industrial Layout, Rivers State, Nigeria. Data of the product variables in the industry, the quantity of raw material used daily for each variable, the quantity of raw material available as well as the mix proportion of the basic raw materials for the production of soap constituted the requisite production data obtained. Also, data were collected on the cost and selling prices of the products. The data consisted of the number of raw materials available for the daily production of five (5) different brands of soap [two (2) laundry soaps and three (3) toilet soaps] and profit contribution per each unit brand of soaps produced.

2.2 Method

We adopted the linear programming maximization tool in our application. The method was facilitated with LINDO software.

2.2.1 Linear Programming Technique

Linear programming is a branch of mathematical programming which is designed to solve optimization problems where objectives and all the constraints involved can be expressed as a linear function. It is a powerful tool in management science and operations research for decision-making under certainty. Linear programming can also be used for verification and checking mechanisms to ascertain the accuracy and the reliability of the decisions which are taken solely based on the manager's experience without the aid of a mathematical model (Naveena *et al.*, 2019). It is useful in the allocation of scarce resources like materials, machines, man, time, etc. To facilitate and accelerated the completion of the calculation of linear programming mathematical model for production optimization is assisted by using a computer program (software) application. There is quite many few software for linear programming simulation such as LiPs, GAMS, CPLEX, MOSEK, TORA, QSB, and LINDO to mention but a few (Sharma, 2009). However, this research will employ the LINDO software for production optimization in the soap production industry.

2.2.2 Linear Programming Model for Soap Production Optimization

The general linear programming model with n decision variables and m constraints can be stated in the following form (Sulaimon *et al.*, 2014):

$$\text{Optimize (Max) } Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n \quad (3.1)$$

Subject to:

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n (\leq = \geq) b_1 \quad (3.2)$$

$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n (\leq = \geq) b_2 \quad (3.3)$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n (\leq = \geq) b_m \quad (3.4)$$

The model (3.1) to (3.4) can also be expressed in a compact form as in (3.5a):

$$\text{Optimize (max) } Z = \sum_{j=1}^n c_j x_j \dots \dots \dots (\text{objective function}) \quad (3.5a)$$

Subject to the linear constraints:

$$\sum_{j=1}^n a_{ij} x_j (\leq, =, \geq) b_i \quad i = 1, 2, \dots, m \quad (3.5b)$$

and

$$x_j \geq 0, \quad j = 1, 2, \dots, n \quad (3.5c)$$

Where, c_1, c_2, \dots, c_n is the per unit profit (or cost) of decision variables x_1, x_2, \dots, x_n to the value of the objective function. b_i is the total availability of the i th resource. Z is the measure – of – performance which can be either profit or cost or reverence etc. And $a_{11}, a_{12}, a_{2n}, a_m, a_{m2}, \dots, a_{mn}$ is the amount of resource consumed per unit of the decision variables.

2.2.3 Standard Form of a Linear Programming Model

The use of the simplex method to solve a linear programming problem requires that the problem be converted into its

standard form. The standard form of a linear programming problem has the following properties:

- i. All the constraints should be expressed as equations by adding slack or surplus variables.
- ii. The right-hand side of each constraint should be made of non-negative (if not). This is done by multiplying both sides of the resulting constraints by -1.
- iii. The objective function should be of a maximization type.

For n decision variables and m constraints, the standard form of the linear programming model can be applied as presented in the model (3.6) to (3.10):

$$\text{Optimize (Max) } Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n + 0s_1 + 0s_2 + \dots + 0s_m \quad (3.6)$$

Subject to the linear constraints

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1 \quad (3.7)$$

$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n = b_2 \quad (3.8)$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n + s_m \geq 0 \quad (3.9)$$

This can be stated in a more compact form:

$$\text{Optimize (max) } Z = \sum_{j=1}^n c_j x_j + \sum_{i=1}^m 0s_i \quad (3.10a)$$

Subject to the linear constraints

$$\sum_{j=1}^n a_{ij} x_j s_i = b_i, \quad i = 1, 2, \dots, m \quad (3.10b)$$

and

$$x_j, s_i \geq 0 \quad (\text{for all } i \text{ and } j) \quad (3.10c)$$

2.2.4 Assumptions Considered for the Model

- i. It is assumed that the raw materials required for the production of soap are limited (scarce).
- ii. It is assumed that an effective allocation of raw materials to the variables will aid optimal production and at the same time maximise the profit of the soap industry.
- iii. It is assumed that the qualities of raw materials used in soap production are standard (not inferior).

2.2.5 Lindo Software

LINDO (Linear, Interactive, and Discrete Optimizer) is a software package tool and it is a convenient, but powerful tool for solving linear, integer, and quadratic programming problems. These problems occur in areas of business, industry, research and government. Specific application areas where LINDO has proven to be of great use would include product distribution, ingredient blending, production and personnel scheduling, and inventory management. The list could easily occupy the rest of this help file. The guiding design philosophy for LINDO has been that, if a user wants to do something simple, there should not be a large setup cost to learn the necessary features of LINDO. At the other extreme, LINDO has been used to solve real industrial linear, quadratic, and

integer programs of respectable size. For commercial applications, LINDO is frequently used to solve problems with tens of thousands of constraints and hundreds of thousands of variables. There are three basic styles of using the LINDO software. For small to medium-sized problems, LINDO is simple to use interactively from the keyboard. Entering a model is quite easy to do. It's also possible to use LINDO with files created elsewhere, containing scripts of commands and input data, and producing files for reporting purposes. Finally, custom-created subroutines may be linked directly with LINDO to form a solution containing both your code and the LINDO optimization libraries. The LINDO software is designed to be simple to learn and use. This is particularly true for small problems. LINDO software (version 2.0) is an algorithm, that is, a mathematical set of programs (mathematical software). It is an optimization system in the area of operations research. LINDO software (version 2.0) provides linear programming solutions that will be useful and available to the management of production industries in reducing the wastage of their production system resources and set-up times which consequently leads to reduced expenses. The study will also assist manufacturing industries to determine an optimal product mix for their production process flow line that will produce the most profit after sales.

2.2.6 Model Simulation

Figure 3.3 illustrates a flowchart showing the step-in applying LINDO software for soap production optimization.

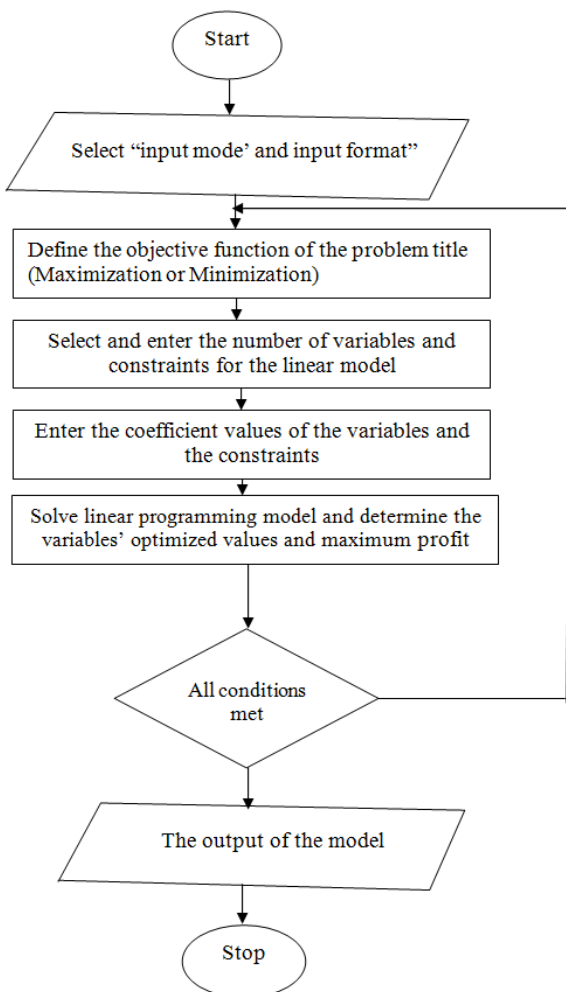


Figure 2.3. Flowchart showing the Steps in applying LINDO Software for Soap Production Optimization (Lindo, 2003)

The linear equations were inputted and programmed into the LINDO software (version 2.0) as follows (LINDO Systems Inc., 2003):

- i. LINDO software (version 2.0) is menu-driven and widows-based which makes it very user-friendly.
- ii. LINDO software (version 2.0) software offers solutions to various models in operational research, but for this study, the 'linear programming model is selected.
- iii. Then the 'input mode' and 'input format' of the linear program be selected and encoded.
- iv. After which, the objective function of the problem title (Maximization or Minimization) is selected and entered.
- v. The number of variables and constraints in the linear model is also selected and entered.
- vi. The coefficient values of the five variables and the seven constraints in the linear program are modelled into the software and saved in the computer database.
- vii. After entering the coefficient values of the variables in the maximization equation and the coefficient values of the constraint's equations, then 'Solve Problem' is selected.
- viii. After the 'Solve Problem' is selected, 'iteration commences if all the criteria for the model simulation are met producing the results as an output of the model'. If not, the process returns to.

3.0 RESULTS AND DISCUSSION

3.1 Investigation of the Current Quantity of Materials for Maximum Profit in Soap Production

The first objective was to investigate the current quantity of materials for maximum profit in soap production. The quantities of raw materials (sodium hydroxide or caustic soda, sodium silicate, perfumes, dye or colour, Palm kernel oil, and salt) available currently for daily production of each of the five (5) different types of soap (Bar Soap, Aura Soap, Medicated Soap, Translucent Soap and Jumbo-all-purpose Soap) was investigated and analysed.

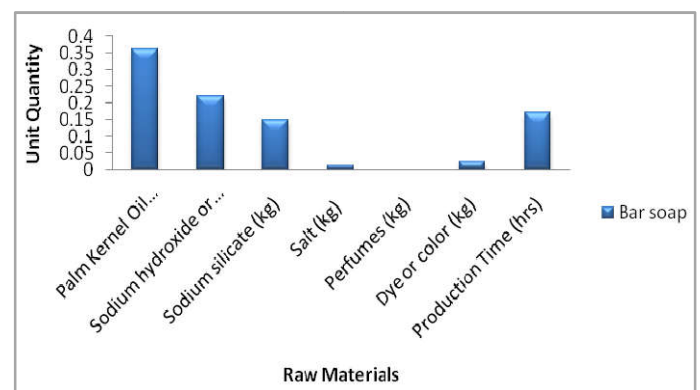


Figure 3.1. Quantities of Raw Materials for Bar Soap

Figure 3.1 illustrates the quantities of raw materials for the production of bar soap in the RIVOC Soap industry. The illustration revealed that the bar soap produced by the company used 0.36kl of Palm Kernel Oil /other Fatty Matter, 0.22kg of Sodium hydroxide (caustic soda), 0.15kg of Sodium silicate, 0.012kg of salt, 0.0005kg of perfumes, and 0.025kg of dye daily. This result is in good agreement with results obtained in studies reviewed in the literature (Balogun *et al.*, 2013; Okolie *et al.*, 2014).

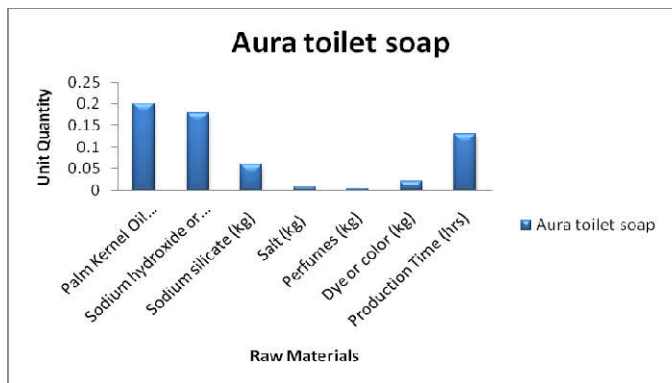


Figure 3.2. Quantities of Raw Materials for Aura Toilet Soap

Figure 3.2 illustrates the quantities of raw materials for the production of Aura toilet soap in the RIVOC Soap industry. The illustration revealed that the Aura toilet soap produced by the company used 0.20kl of Palm Kernel Oil /other Fatty Matter, 0.18kg of Sodium hydroxide or caustic soda, 0.06kg of Sodium silicate, 0.008kg of salt, 0.0004kg of perfumes, and 0.02kg of dye daily. This result correlates with results obtained in the research reviewed in the literature (Balogun *et al.*, 2013; Okolie *et al.*, 2014).

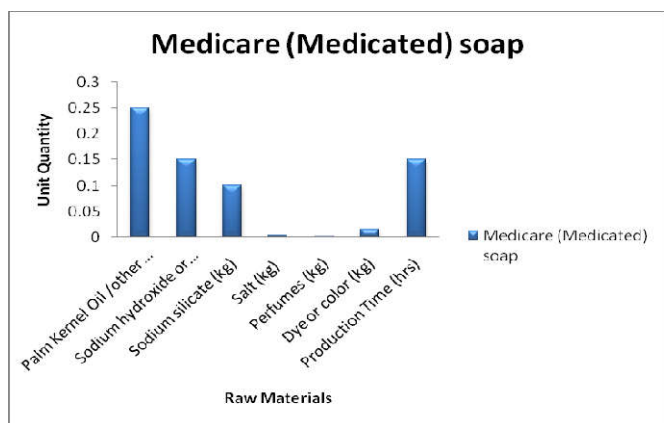


Figure 3.3. Quantities of Raw Materials for Medicare Medicated Soap

Figure 3.3 illustrates the quantities of raw materials for the production of Medicare medicated soap in the RIVOC Soap industry. The illustration revealed that the Medicare medicated soap produced by the company used XC0.25kl of Palm Kernel Oil /other Fatty Matter, 0.15kg of Sodium hydroxide or caustic soda, 0.10kg of Sodium silicate, 0.005kg of salt, 0.0007kg of perfumes, and 0.015kg of dye daily. This result agrees with results obtained in studies reviewed in the literature (Balogun *et al.*, 2013; Okolie *et al.*, 2014).

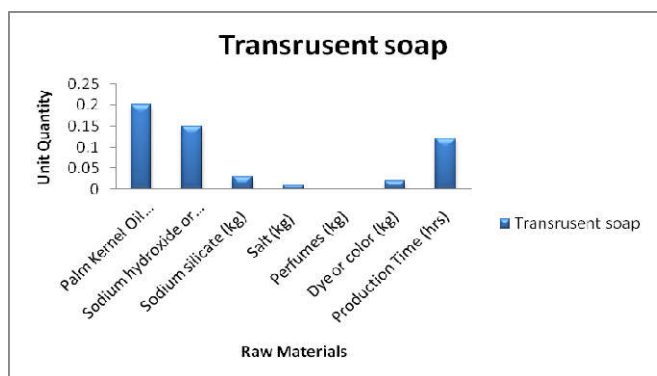


Figure 3.4. Quantities of Raw Materials for Translucent Soap

Figure 3.4 illustrates the quantities of raw materials for the production of Translucent soap in the RIVOC Soap industry. The illustration revealed that the Translucent soap produced by the company used 0.20kl of Palm Kernel Oil /other Fatty Matter, 0.15kg of Sodium hydroxide or caustic soda, 0.03kg of Sodium silicate, 0.010kg of salt, 0.0009kg of perfumes, and 0.020kg of dye daily. This result is in good agreement with results obtained by Balogun, *et al.* (2013) and Okolie *et al.* (2014).

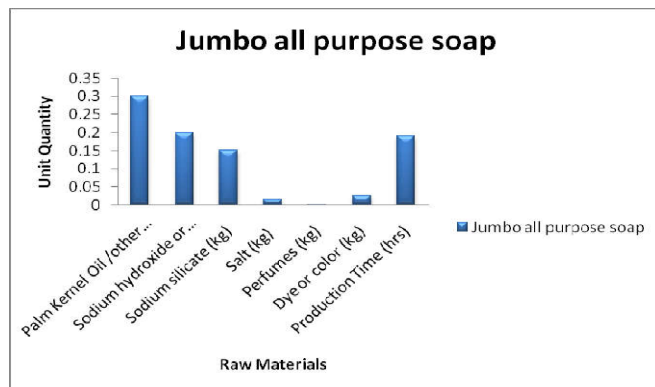


Figure 3.5. Quantities of Raw Materials for Jumbo-all-purpose Soap

Figure 3.5 illustrates the quantities of raw materials for the production of Jumbo-all-purpose soap in the RIVOC Soap industry. The illustration revealed that the Jumbo-all-purpose soap produced by the company used 0.30kl of Palm Kernel Oil /other Fatty Matter, 0.20kg of Sodium hydroxide or caustic soda, 0.15kg of Sodium silicate, 0.015kg of salt, 0.001kg of perfumes, and 0.025kg of dye daily. This result is in good agreement with results obtained in studies reviewed in the literature (Okolie *et al.*, 2014; Balogun, *et al.*, 2013).

3.2 Understudying the Time Taken to Produce the Different Variables of Soap

The second objective was to understudy the time taken to produce each of the different variables (types) of soap. The time taken to produce each of the different variables (types) of soap (bar soap, aura soap, medicated soap, Translucent soap and jumbo-all-purpose soap) was understudied and analysed. The total amount of production time available for manufacturing daily was 20 hours.

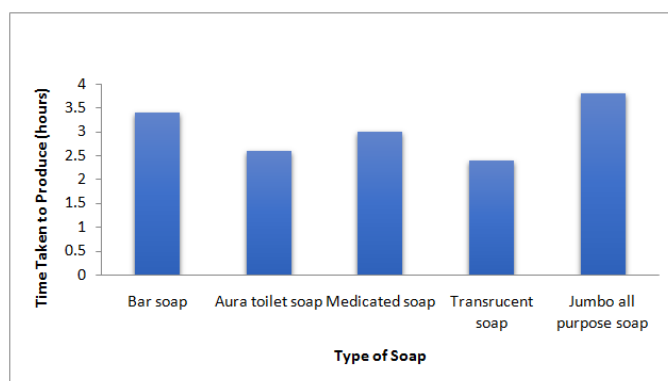


Figure 3.6. Time Taken to Produce the Different Soap Variables

Figure 3.6 illustrates the time taken to produce the different variables of soap (Bar Soap, Aura Soap, Medicated Soap, Translucent Soap and Jumbo-all-purpose Soap). The

illustration showed that the time taken to produce the Bar soap daily is 3.4hrs i.e. 17% of the total available time for production daily, and the time taken to produce the Aura soap daily is 2.6hrs i.e. 13% of the total available time for production daily, the time taken to produce the Medicated soap daily is 3.0hrs i.e. 15% of the total available time for production daily, the time taken to produce the Translucent soap daily is 2.4hrs i.e. 12% of the total available time for production daily, and the time taken to produce the Jumbo-all-purpose soap daily is 3.8hrs i.e. 19% of the total available time for production daily. The result showed that Jumbo-all-purpose soap used the greatest amount of time for production while Translucent soap used the least amount of time for production. The average cost price in Naira, the average selling price in Naira and the profit contribution from each unit of the different types of soap produced is presented in Appendix A3.

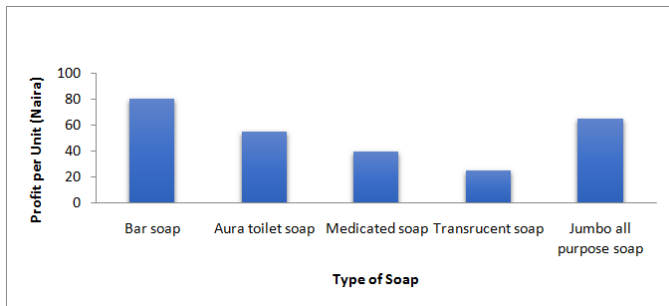


Figure 3.7. Profit Contribution per unit Soap Product

Figure 3.7 illustrates the profit contribution per unit of the different variables of soap (Bar Soap, Aura Soap, Medicated Soap, Translucent Soap and Jumbo-all-purpose Soap). The illustration showed that each unit of bar soap contributes a profit of ₦80 after sales, each unit of Aura toilet soap contributes a profit of ₦55 after sales, each unit of Medicated soap contributes a profit of ₦40 after sales, each unit of Translucent soap contributes a profit of ₦25 after sales and each unit of Jumbo-all-purpose soap contributes a profit of ₦65 after sales. The result shows that each unit of bar soap contributes the greatest profit while the Translucent soap contributes the least to the soap industry in terms of profit.

3.3 Determination and Optimization of the Product Mix and Schedule of Raw Materials in Soap Production for Maximum Profit using Linear Programming

The third objective was to determine the product mix and schedule of raw materials in soap production for maximum profit using linear programming.

- Let the quantity of bar soap to be produced = x_1
- Let the quantity of Aura toilet soap to be produced = x_2
- Let the quantity of Medicated soap to be produced = x_3
- Let the quantity of Translucent soap to be produced = x_4
- Let the quantity of Jumbo-all-purpose soap to be produced = x_5

The results for the first, second and third iteration is simulated and presented in Table 5, 6 and 7 respectively in Appendix A. the algorithm for the calculation of the first iteration elements

is presented in Appendix A5 while the algorithm for the calculation of the second iteration elements is presented in Appendix A6 and the algorithm for the calculation for the third iteration elements is presented in Appendix A7.

Answer:
 $F^* = 68421.05$
 $X^* = (0; 0; 0; 0; 1052.63)$

The formulated linear programming model was solved using LINDO software, and the resulting output from the LINDO computer simulation is shown in Figure 4.8. The third iteration variant gives an optimal solution of: $x_1 = 0.01, x_2 = 0.0, x_3 = 0.0, x_4 = 0.0, x_5 = 1052.63$ and $F = 68,421.05$. Hence, for the third objective of this work, the optimized product mix and schedule of raw material determined for maximum profit in soap production is 0.01 units of bar soap, 0 units of Aura toilet soap, 0 units of Medicated soap, 0 units of Translucent soap, and 1052.65 unit of Jumbo-all-purpose soap, The third iteration showed that using the optimized product mix the maximum profit that could be realized daily in Rivers State Vegetable Oil Company is ₦68,421.05 and producing 1052.65 units of Jumbo-all-purpose soap guarantees this maximum profit.

4.0 CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

From the results and outcome of this study, the following conclusions were made:

The current quantity of materials for maximum profit in soap production is useful as a result of the different amounts of raw materials contained in each soap produced daily. The time taken to produce the different variables of soap was understudied. Bar soap used 17% of the available production time daily, Aura toilet soap used 13% of the available production time daily, Medicated soap used 15% of the available production time daily, Translucent soap used 12% of the available production time daily and Jumbo-all-purpose soap used 19% of the available production time daily. The product mix and schedule of raw materials in soap production for maximum profit were determined using linear programming. The results for the optimal product mix and schedule of raw material determined for variables in the soap production industry for profit maximization is 0.01 unit of bar soap, 0 unit of Aura toilet soap, 0 unit, 0 unit of Medicated soap, and 1052.65 unit of Jumbo-all-purpose soap, and course, the maximum profit that could be realized daily in Rivers State Vegetable Oil Company is ₦68,421.05 and the 1052.65 units of Jumbo-all-purpose soap guarantees this maximum profit based on the data collected the optimum result derived from the model indicates that he Jumbo-all-purpose soap should be produced daily, for optimum profit. The daily production quantities for the jumbo soap should be 1052.63 units which produced a maximum profit of ₦68,421.05 daily. Based on the analysis carried out in this research work and the result shown, RIVOC Soap Production Industry should produce five different types of soap to satisfy its customers, but more Jumbo-all-purpose soap should be produced to attain maximum profit because they contribute mostly to the profit earned by the company.

4.2 Recommendations

Based on the results and findings in this study, the following recommendations are made to advocate effective production using raw material optimization in manufacturing firms:

The Government of the Federal Republic of Nigeria should assist in making provisions for grants and funds for Universities and scholars to facilitate more research on the application of artificial intelligence for production management in the manufacturing sector, as finance was a major limitation of this study. We further recommend that linear programming with LINDO optimization software for profit maximization should be encouraged in other production industries in Nigeria as linear programming could be applied in manufacturing firms having multiple products or service operations with more than one type of service at a time.

4.3 Contribution to Knowledge

This research has contributed to the pool of knowledge in the following areas:

The research has enabled the determination of soap production and the time taken to produce the types of soap for maximum profit and it was discovered that the type of soap which required the most time to produce, is the variable (type) with the most constrained to time that guarantees maximum profit.

The linear programming modelling and simulation with the LINDO optimization tool that was employed in the study was effective in determining the product mix and schedule of raw materials in soap production for maximum profit as evidently, it was discovered that the five different soap products should be produced to satisfy her customers, but the variable most constrained to time (Jumbo-all-purpose soap) should be greatly produced for profit maximization.

5. Acknowledgements

The authors express profound gratitude to Engr. Prof. B. T. Lebele Alawa, (the HOD Mechanical Engineering Department), all departmental and faculty of engineering Lecturers.

REFERENCES

- Aadharshana, V. M. (2019). Resource Optimization in Construction. *International Journal of Engineering Science and Computing*, 9(11), 24019 – 24022.
- Abiodun, A. J. (2011). Quantitative Analysis of Efficiency of Public Health Care Facilities in Nigeria. Unpublished Doctoral Thesis, Covenant University, Ota.
- Adaku, U. & Melody, M. (2013). Soap Production Using Waste Materials of Cassava Peel and Plantain Peel Ash as an Alternative Active Ingredient, Implication for Entrepreneurship. *International Organization of Scientific Research (IOSR) Journal of Very Large-Scale Integration (VLSI) and Signal Processing*, 3(3), 2319 – 4197.
- Adegbuyi, P. A. O. & Asapo, E. S. (2010). The Effect of Production Planning and Budgeting on Organizational Productivity. *Leonardo Electronic Journal of Practices and Technologies*, 16(4), 201-217.
- Akpan, N. P. & Iwok, I.A. (2016). Application of Linear Programming for Optimal Use of Raw Materials in Bakery. *International Journal of Mathematics and Statistics Invention*, 4(8), 51-57.
- Allahverdi, A. Gupta, J. N. D. & Aldowaisan, T. (1999). A Review of Scheduling Research involving Setup Considerations. *International Journal of Management Science*, 27(4), 219-239.
- Amole, B. B., Adebisi, S. O. & Osuolale, O.M. (2016). Production Planning in the Nigerian Detergent Producing Firm: A Linear Programming Method. *Fountain University Journal of Management and Social Science*, 5(1), 13-24.
- Andre, T. (2010). Optimal Short-Term Hydro Scheduling from the Principle of Progressive Optimality, Published by *American Geophysical Union*, 17(3), 481 – 486.
- Andreas, W. & Herbert, J. (2007). Production Scheduling Problems, *International Journal of Management Science*, 8(2), 1 – 30.
- Asifs, R. (2008). A Comparative Study of Heuristic Algorithms on Economic Lot Scheduling Problem, *International Conference on Engineering and Management Technology*, New York, 55(1), 94 – 109.
- Antezana, W., Calve, S., Beccaccia, A., Ferrer, P., Blas, C. D., Rebollar, P. G. & Andi, S. A., Hendra, P. & Suci, S. (2021). Revenue Optimization Analysis of Rattan Furniture Industry in Palu City, Central Sulawesi, Indonesia (Case Study in PT Meubel Rotan Subur). *Proceedings of the Joint Symposium on Tropical Studies*, 11(10), 58-64.
- Bai, S. & Gershwin, S. (1994). Collaborative Networked Organizations Concepts and Practices in Manufacturing Enterprises, *Computer & Industrial Engineering Journal*, 32(2), 365-385.
- Balasubramanian, J. & Grossman, I. E. (2003). Scheduling Optimization under Uncertainty, *Journal of Computers and Chemical Engineering*, 27(8), 469-490.
- Balogun, O. S., Olatunji, A. A. & Momoh, A. A. (2013). An Optimization Procedure in a Production Line of Sokat Soap Industry, Ikotun, Lagos State. *International Journal of Engineering Research Technology*, 7(2), 50-54.
- Bambang, S. A., Rosida, R. M., Anggun, M. M., Cindy, D. N. & Iit, Y. (2019). Profit Optimization Using Simplex Methods on Home Industry, Bintang Bakery in Sukarame Bandar Lampung. *Institute of Physics (IOP) Conf. Series: Journal of Physics: Conf. Series*, 10(10), 141-153.
- Banjoko, S. A. (2002). Production and Operations Management. *Journal of Construction Engineering and Management*, 112 (1), 14-21.
- Berrichi, A., Yalaoui, F. Amodeo, L. & Mezghiche, M. (2010.) An Optimal Production Planning and Maintenance Policy for a Multiple-Product and Single Machine under Failure rate Dependency. *Computers and Operations Research*, 37(9), 1584 – 1596.
- Biskup, D. (1999). On Scheduling around Large Restriction Common due to Window. *European Journal of Operational Research*, 115(1), 173 – 178.
- Buffa, E.S. (2001). *Production and Operations Management*. 8th edition New York: Wiley Publishing.
- Benedict, I.E. & Uzochukwu, A. (2012). Optimizing Profit with the Linear Programming Model: A Focus on Golden Plastic Industry Limited, Enugu, Nigeria. *Interdisciplinary Journal of Research in Business*, 2(2), 37–49.
- Camarinha-Matos, L. M., Afsarmanesh, H., Galeano, N. & Molina, A. (2009). Collaborative Networked Organizations Concepts and Practice in Manufacturing Enterprises, *Journal of Computers and Industrial Engineering*, 57(1), 46 – 60.

- Cerisuelo, A. (2015). Effects of Nutrition on Digestion Efficiency and Gaseous Emissions from Slurry in Growing Pigs: III. Influence of varying the Dietary Level of Calcium Soap of Palm Fatty Acids Distillate with or without Orange Pulp Supplementation. *Animal Feed Science and Technology*, 209(10), 128-136.
- Chaku, S. E., Gabriel, F. G., Bello, Z. R., Maijama, B. & Adehi, M. U. (2020). Raw Materials Optimization in a Bread Making Factory: An R Implementation. *The International Journal of Business & Management*, 8(8), 361-367.
- Cheng, R., Mitsuo, G. & Yasuhiro, T. (1996). A Tutorial Survey of Job-Shop Scheduling Problems Using Genetic Algorithms. *Computers and Industrial Engineering*, 30(4), 983 – 997.
- Christian, B., Dirk, C. & Mattfeld, B. (1996). Production Scheduling and Rescheduling with Genetic Algorithms, *Evolutionary Computation*, 7(1), 1-17.
- Denys, F. (2006). Applications of Linear Programming. Case Study, Minimizing the Cost of Transportation. Unpublished Master's Degree Dissertation, School of International Business, Arcada.
- Doganis, H. S. (2008). *Annals of Operations Research*, Publisher: Springer Science +Business Media, Athens, 159(1), 315-331.
- Doganis, P. & Sarimveis, H. (2008). Optimal Production Scheduling for the Dairy Industry. *Animal Operation Research*, 159(1), 315-331.
- Dong-Won, K., Kyong-Hee, K., Wooseung, J. & Frank, C. (2002). Unrelated Parallel Machine Scheduling with Setup Times Using Simulated Annealing. *Robotics and Computer Integrated Manufacturing*, 18(2), 223 – 231.
- Goutham, R. (2017). Production Planning in Different Stages of a Manufacturing Supply Chain Under Multiple Uncertainties. Unpublished M.Sc. Thesis, Iowa State University, Ames, Iowa.
- Guerriero, F. (2008). Optimal Theory Application. Published by Springer Science + Business Media Madrid, *Journal of Physical Chemistry*, 2(8), 864-851.
- Habibollah, J. & Kianehkandi, M. (2011). Optimal Scheduling in a Milk Production Line Based on Mixed Integer Linear Programming. 2nd International Conference on Education and Management Technology, IPEDR, Arak, Singapore, 13(11), 86-95.
- Harvey, J. G. (2020). The Role of Software in Optimization and Operations Research. *Encyclopedia of Life Support Systems*, 2(1), 1-11.
- Higgins, J. M. (2001). *Strategic Management and Operations*. Chicago: The Dryden Press.
- Hong-Sen, Y., Qi-Feng, X., Min-Ru, Z. & Xia-Ling, L. (2003). Integrated Production Planning and Scheduling on Automobile Assembly Lines, *International Journal of Production Research*, 35(8), 711-725.
- Hwang, C. L., Fan, L. T., Tiunan, F. A. & Sharma, R. (1970). Optimal Production Planning and Inventory Control, *International Journal of Production Research*, 8(1), 75 – 83.
- Isaac, A. (2015). Optimization of Production Scheduling as Linear Programming Model. *Masters's Degree Thesis Submitted to the Department of Mathematics*, Kwame Nkrumah University Of Science and Technology, Kamasu.
- Ikon, M. A. & Nwankwo, C. N. (2016). Production Planning and Profitability of Selected Manufacturing Firms in Nigeria. *International Journal of Business and Management Review*, 4(1), 11-32.
- Jain K. C. & Aggarwal, L. N. (2008). *Production Planning, Control and Industrial Management*. Delhi, Nai-Sarak: Khalma Publishers.
- Karmaker, C. L. & Halder, P. (2017). Scheduling Project Crashing Time Using Linear Programming Approach: Case Study. *International Journal of Research in Industrial Engineering*, 6(4), 283-292.
- Khmelnitsky, E., Konstantin, K. & Oded, M. (1993). Optimal Control of Production Scheduling, Academic Publishers, Boston.
- Khmelnitsky, E. (2002). Optimal Control Approach to Production Systems with Inventory. *International Journal of Research in Industrial Engineering*, 47(2), 289 – 292.
- Klaus, J., Roberto, S. O. & Maxim, S. (2010). A Linear Time Approximation Scheme for Job Shop Scheduling. *European Journal of Operational Research*, 167(2), 297-319.
- Kumar, S. A. & Suresh, N. (2008). *Production and Operations Management* (Second Edition). New Age International (P) Limited, New Delhi.
- Lei, D. (2008). *Multi-objective Production Scheduling*. Springer-Verlag-London Limited, 926 – 938.
- LINDO Systems Inc. (2003). *LINDO Computer Software User's Handbook*, Revised Edition, Illinois, Lindo Systems Inc. Publishers, Chicago.
- Li-Ning, X., Ying-Wu, C. & Ke-Wei, Y. (2009). Multi-objective Flexible Job Shop Schedule. *International Journal of Applied Operational Research*, 9(1), 362 – 376.
- Lombardi, M. & Michella, M. (2012). Optimal Methods for Resource Allocation and Scheduling, Springer Link, 17.
- Maravelias, C. T. & Sung, (2008). Integration of Production Planning and Scheduling: Overview challenges and opportunities. *Proceeding Foundation of Computer Aided Process Operations (FCAPO)*.
- Miguel, A., Gonzalez, C. R., Vela, R. & Ramiro, V. (2006). Genetic Algorithm combined with Tabu Search for the Job Shop Scheduling Problem with Setting up Times, *International Journal of Production Economics*, 132(2), 279 – 291.
- Min Huang, R., Bo, S. & Linrong, L. (2013). Research on the Production scheduling *Optimization for Virtual Enterprises*, 12(4), 112-123.
- Musikapun, P. & Pupong, P. (2012). Solving Multi-Stage, Multi Machine, and Multi-Product Scheduling Problems using BAT Algorithm. *International Journal of Applied Operational Research*, 11(4), 35-41.
- Naderi, B., Zandich, M., Khaleghi, A., Ghoshe, B. & Roshanaei, V. (2009). *Expert Systems with Applications*, 36(6), 9625-9633.
- Naveena, J.D., Surya, K. P. R. & Sivasundari, M. (2019). Application of Linear Programming Model for Production on Planning in an Engineering Industry-A Case Study. *International Journal of Engineering Research & Technology*, 8(10), 37-52.
- Ndubuisi, C. O., Pius, C. O., Nnaemeka, O. & Gracefield, O. R. I. (2012). Modelling of Production Plan and Scheduling of Manufacturing Process for a Plastic Industry in Nigeria. *Industrial Engineering Letters*, 2(8), 24-35.
- Nsikan, P. A. & Okeh, C. O. (2017). Karmarkar's Approach for Solving Linear Programming Problem for Profit Maximization in Production Industries: NBC Port-Harcourt Plant. *American Journal of Statistics and Probability*, 2(1), 1-8.
- Okeke, S. U. N. (2009). *Home Economics for Schools and Colleges*, Onitsha: Africana First Publishers Plc.

- Okolie, C. P., Ezeliora, C. D., Iwenofu, C. O. & Sinebe, J. E. (2014). Optimization of a Soap Production Mix Using Response Surface Modeling: A Case of Niger Bar Soap Manufacturing Industry Onitsha, Anambra State, Nigeria. *International Journal of Scientific & Technology Research*, 3(9), 346-352.
- Ovunda, M. N., Isaac, O. E. & Ndor, M. V. (2019). Assessment of the Impact of Production Planning and Operational Cost Control in the Beverage Industries. *American Journal of Engineering Research*, 8(1), 63-81.
- Omboga, J.K. & Okibo, W. B. (2016). Effects of Financial Planning Practices on the Growth of Small Manufacturing Firms in Kisii County, Kenya. *International Journal of Economics, Commerce and Management*, 4(4), 1224-1233.
- Olamade, O. & Oni, O. (2016). Manufacturing and Economic Growth in Africa: A Panel Test of Kaldor's First Growth Law. *Journal of Economics and Sustainable Development*, 7(22), 126 – 140.
- Parthiban, P., Punniyamoorthy, M., Ganesh, K. & Dominic, P. D. D. (2010). An Integrated Model for Optimization of Production-Distribution Inventory Levels and routing Structure for a multi-period, multiproduct, bi-echelon supply chain. *The International Journal of Applied Management and Technology*, 6(1), 153-178.
- Phanseil, O. N., Dueno, E. & Xianghong, W. Q. (1998). Synthesis of Exotic Soaps in the Chemistry Laboratory. *Journal of Chemistry Education*, 75(5), 612-631.
- Richard, L. & Wei, Z. (1999). Computers and Industrial Engineering, published by: Elsevier Ltd, 37(1-2), 57 -61.
- Rivers Vegetable Oil Company Limited (RIVOC) (2006a). *Company Profile*, Artsaels Production Limited, Port Harcourt.
- Rivers Vegetable Oil Company Limited (RIVOC) (2006b). *Product Brochure*. <http://www.rivoc.com> Accessed, 13th February 2020.
- Sameh, A. S. E. & Nahid, S. (2019). Waste reduction by Linear Programming Optimizing. *18th International Conference on Aerospace Sciences & Aviation Technology*, Cairo.
- Sharma, J. K (2009). *Operations Research, Theory and Applications*, Macmillan Publishers Ltd, Delhi.
- Sinebe, J. E., Okonkwo, U. C. & Enyi, L. C. (2014). Simplex Optimization of Production Mix: A Case of Custard Producing Industries in Nigeria. *International Journal of Applied Science and Technology*, 4(4), 180-189.
- Solaja, O., Abiodun, J., Abioro, M. Ekpudu, J. & Olasubulumi, O. (2019). Application of Linear Programming Techniques in Production Planning. *International Journal of Applied Operational Research*, 9(3), 11-19.
- Sonke, H., Wei, Z. & Dirk, B. (2010). A survey of Variants and Extensions of the Resource-Constrained Project Scheduling Problem. *European Journal of Operational Research*, 207(1), 1 – 14.
- Sulaimon, O. A., Bilqis, B. A. & Ismail, O. S. (2014). Linear Optimization Techniques for Product-Mix of Paints Production in Nigeria. *Journal of Operations Research; Statistical Decision Theory*, 10(1), 181-190.
- Tuncel-G., Miguel, G. & Mirac, B. (2007) Applications of Petri nets in Production scheduling, Springer-Verlag London Limited, 34(2), 762 – 773.
- Tvakkoli-Moghadam, R. & Daneshmand-Mehr, M. (2005). A Computer Simulation Model for Job Shop Scheduling Problems Minimizing Makespan, *Journal of Computer and Industrial Engineering*, 48(4), 811 – 823.
- Umoh, G. I., Ify, H. W. & Edwinah, A. (2013). Production Planning and Corporate Productivity Performance in the Nigerian Manufacturing Industry. *IOSR Journal of Business and Management*, 14(2), 1-7.
- Vollman, T. E., Berry, W. L. & Why Bark, D.C. (2007). *Manufacturing Planning and Control Systems*, Boston: McGraw-Hill.
- Warra, A. A. (2013). Soap Making in Nigeria using Indigenous Technology and Raw Materials. *African Journal of Pure and Applied Chemistry*, 7(4), 139-145
- Wild, R. (2008). *Production and Operations Management*, New York: Holt, Risehart and Winston.
- Yan, H. S. & Zhang, X.D. (2007). A Case Study on Integrated Production Planning and Scheduling in a Three-Stage Manufacturing System. *IEEE Transaction on Automatic Science and Engineering*, 4(2), 86-92.
- Yasser, M.R. & Aboelmagd, M. (2018). Linear Programming Applications in Construction Sites. *Alexandria Engineering Journal*, 57(31), 4177-4187.
- Zhang, G., Shao, X., Li, P. & Gao, L. (2009). An Effective Hybrid Particle Swarm Optimization Algorithm for Multi-Objective Flexible Job Shop Scheduling Problem, *International Journal of Research in Industrial Engineering*, 56(4), 1309 – 1318.
- Zobolas, G. I., Tarantilis, C. D. & Loannou, G. (2009). Minimizing Makespan in Permutation Flow Shop Scheduling Problem, *Computers and Operation Research*, 36(4), 1249 -1267.

Table A 6. Second Iteration Results (Iteration: 2)

B	Cb	P	x ₁	x ₂	x ₃	x ₄	x ₅ ↓	x ₆	x ₇	x ₈	x ₉	x ₁₀	x ₁₁	x ₁₂	Q
			80	55	40	25	65	0	0	0	0	0	0	0	
x ₆	0	1457.65	0	-0.08	-0.07	-0.05	0.26	1	0	0	0	0	0	-2.12	5611.41
x ₇	0	1174.12	0	0.01	-0.04	-0.01	0.18	0	1	0	0	0	0	-1.29	6693.49
x ₈	0	282.35	0	-0.05	-0.03	-0.08	0.13	0	0	1	0	0	0	-0.88	2119.21
x ₉	0	998.59	-0	0	-0.01	0	0.01	0	0	0	1	0	0	-0.07	73109.39
x ₁₀	0	499.94	0	0	0	0	0	0	0	0	0	1	0	0	529532.71
x ₁₁	0	1997.06	0	0	-0.01	0	0.02	0	0	0	0	0	1	-0.15	89933.77
x ₁ ←	80	117.65	1	0.76	0.88	0.71	0.11	0	0	0	0	0	0	5.88	1052.63
Max		9411.76	0	6.18	30.59	31.47	-56.06	0	0	0	0	0	0	470.59	

Table A 7. Third Iteration Results (Iteration: 3)

B	Cb	P	x ₁	x ₂	x ₃	x ₄	x ₅	x ₆	x ₇	x ₈	x ₉	x ₁₀	x ₁₁	x ₁₂	Q
			80	55	40	25	65	0	0	0	0	0	0	0	
x ₆	0	1184.21	-2.32	-1.85	-2.12	-1.69	0	1	0	0	0	0	0	0	-15.79
x ₇	0	989.47	-1.57	-1.19	-1.43	-1.11	0	0	1	0	0	0	0	0	-10.53
x ₈	0	142.11	-1.19	-0.97	-1.08	-0.92	0	0	0	1	0	0	0	0	-7.89
x ₉	0	984.21	-0.12	-0.09	-0.11	-0.08	0	0	0	0	1	0	0	0	-0.79
x ₁₀	0	498.95	-0.01	0	-0.01	-0.01	0	0	0	0	0	1	0	0	-0.05
x ₁₁	0	1973.68	-0.2	-0.15	-0.18	-0.14	0	0	0	0	0	0	1	0	-1.32
x ₅	65	1052.63	8.95	6.84	7.89	6.32	1	0	0	0	0	0	0	0	52.63
Max		68421.05	501.58	389.74	473.16	385.53	0	0	0	0	0	0	0	0	3421.05

Appendix B: Algorithms from LINDO Optimization Software for Profit Maximization of Soap Production in Rivers State Vegetable Oil Company

Let F denote the profit to be maximized

The linear programming model for the above production data is given by:

$$Max F = 80x_1 + 55x_2 + 40x_3 + 25x_4 + 65x_5$$

S.t.

$$0.36x_1 + 0.20x_2 + 0.25x_3 + 0.20x_4 + 0.30x_5 \leq 1500$$

$$0.22x_1 + 0.18x_2 + 0.15x_3 + 0.15x_4 + 0.20x_5 \leq 1200$$

$$0.15x_1 + 0.06x_2 + 0.10x_3 + 0.03x_4 + 0.15x_5 \leq 300$$

$$0.012x_1 + 0.008x_2 + 0.005x_3 + 0.010x_4 + 0.015x_5 \leq 1000$$

$$0.0005x_1 + 0.004x_2 + 0.0007x_3 + 0.0009x_4 + 0.001x_5 \leq 500$$

$$0.025x_1 + 0.020x_2 + 0.015x_3 + 0.020x_4 + 0.025x_5 \leq 2000$$

$$0.17x_1 + 0.13x_2 + 0.15x_3 + 0.12x_4 + 0.019x_5 \leq 20$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Converting the model into its corresponding standard forms;

$$Max F = 80x_1 + 55x_2 + 40x_3 + 25x_4 + 65x_5 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6$$

S.t.

$$0.36x_1 + 0.20x_2 + 0.25x_3 + 0.20x_4 + 0.30x_5 + s_1 \leq 1500$$

$$0.22x_1 + 0.18x_2 + 0.15x_3 + 0.15x_4 + 0.20x_5 + s_2 \leq 1200$$

$$0.15x_1 + 0.06x_2 + 0.10x_3 + 0.03x_4 + 0.15x_5 + s_3 \leq 300$$

$$0.012x_1 + 0.008x_2 + 0.005x_3 + 0.010x_4 + 0.015x_5 + s_4 \leq 1000$$

$$0.0005x_1 + 0.004x_2 + 0.0007x_3 + 0.0009x_4 + 0.001x_5 + s_5 \leq 500$$

$$0.025x_1 + 0.020x_2 + 0.015x_3 + 0.020x_4 + 0.025x_5 + s_6 \leq 2000$$

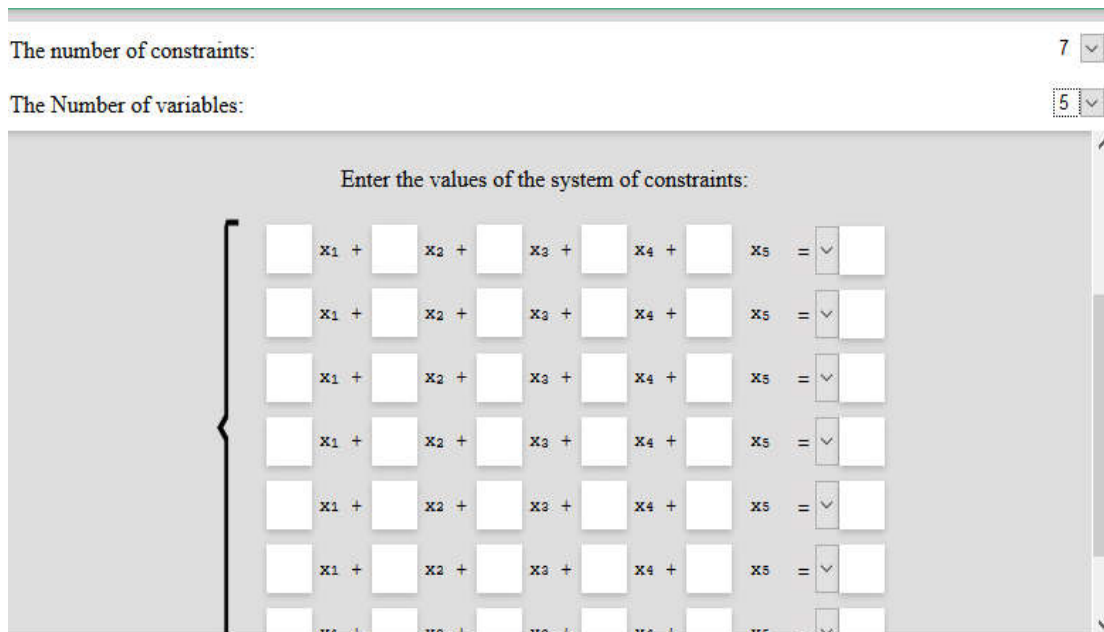
$$0.17x_1 + 0.13x_2 + 0.15x_3 + 0.12x_4 + 0.019x_5 + s_7 \leq 20$$

$$x_1, x_2, x_3, x_4, x_5, s_1, s_2, s_3, s_4, s_5, s_6, s_7 \geq 0$$

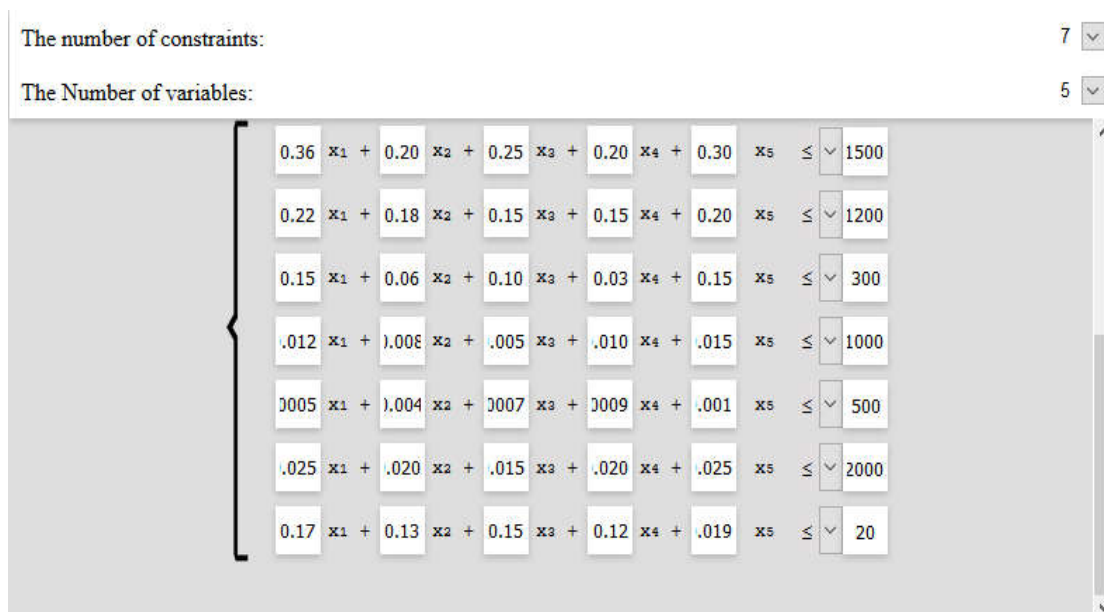
Model Application and Simulation

These linear equations are then inputted and programmed into the LINDO Software as follows:

- i. LINDO is menu-driven and Windows-based which makes it very user-friendly.
- ii. The LINDO software offers solutions to various models in operational research, but for this study, the 'linear programming model is selected.
- iii. Then the 'input mode' and 'input format' of the linear program be selected and encoded.
- iv. After which, the objective function of the problem title (Maximization or Minimization) is selected and entered.
- v. The number of variables and constraints in the linear model is also selected and entered as shown in Figure 4.8.



LINDO Software Interface for number of variables and constraints input



LINDO Software Interface for number of variables and constraints input

- vi. The coefficient values of the five (5) variables and the seven (7) constraints in the linear program are modelled into the software and saved in the computer database as shown in Figure 4.9.
- vii. After entering the coefficient values of the variables in the maximization equation and the coefficient values of the constraint's equations, then 'Solve Problem' is selected.

Calculation of table elements:

V

V

V

Elements of the column basis (B)

Transfer to the table the basic elements that we identified in the preliminary stage:

$$B_1 = x_6;$$

$$B_2 = x_7;$$

$$B_3 = x_8;$$

$$B_4 = x_9;$$

$$B_5 = x_{10};$$

$$B_6 = x_{11};$$

$$B_7 = x_{12};$$

Cb column items

Each cell of this column is equal to the coefficient, which corresponds to the base variable in the corresponding row.

$$Cb_1 = 0;$$

$$Cb_2 = 0;$$

$$Cb_3 = 0;$$

$$Cb_4 = 0;$$

$$Cb_5 = 0;$$

$$Cb_6 = 0;$$

$$Cb_7 = 0;$$

Values of variable variables and column P

At this stage, no calculations are needed, just transfer the values from the preliminary stage to the corresponding table cells:

$$P_1 = 1500;$$

$$P_2 = 1200;$$

$$P_3 = 300;$$

$$P_4 = 1000;$$

$$P_5 = 500;$$

$$P_6 = 2000;$$

$$P_7 = 20;$$

$$x_{1,1} = 0.36;$$

$$x_{1,2} = 0.2;$$

$$x_{1,3} = 0.25;$$

$$x_{1,4} = 0.2;$$

$$x_{1,5} = 0.3;$$

$$x_{1,6} = 1;$$

$$x_{1,7} = 0;$$

$$x_{1,8} = 0;$$

$$x_{1,9} = 0;$$

$$x_{1,10} = 0;$$

$$x_{1,11} = 0;$$

$$x_{1,12} = 0;$$

$$x_{2,1} = 0.22;$$

$$x_{2,2} = 0.18;$$

$$x_{2,3} = 0.15;$$

$$x_{2,4} = 0.15;$$

$$x_{2,5} = 0.2;$$

$$x_{2,6} = 0;$$

$$x_{2,7} = 1;$$

$$x_{2,8} = 0;$$

$$x_{2,9} = 0;$$

$$x_{2,10} = 0;$$

$$x_{2,11} = 0;$$

$$x_{2,12} = 0;$$

$$x_{3,1} = 0.15;$$

$$x_{3,2} = 0.06;$$

$$x_{3,3} = 0.1;$$

$$x_{3,4} = 0.03;$$

$x_{3,5} = 0.15;$
 $x_{3,6} = 0;$
 $x_{3,7} = 0;$
 $x_{3,8} = 1;$
 $x_{3,9} = 0;$
 $x_{3,10} = 0;$
 $x_{3,11} = 0;$
 $x_{3,12} = 0;$
 $x_{4,1} = 0.01;$
 $x_{4,2} = 0.01;$
 $x_{4,3} = 0.01;$
 $x_{4,4} = 0.01;$
 $x_{4,5} = 0.02;$
 $x_{4,6} = 0;$
 $x_{4,7} = 0;$
 $x_{4,8} = 0;$
 $x_{4,9} = 1;$
 $x_{4,10} = 0;$
 $x_{4,11} = 0;$
 $x_{4,12} = 0;$
 $x_{5,1} = 0;$
 $x_{5,2} = 0;$
 $x_{5,3} = 0;$
 $x_{5,4} = 0;$
 $x_{5,5} = 0;$
 $x_{5,6} = 0;$
 $x_{5,7} = 0;$
 $x_{5,8} = 0;$
 $x_{5,9} = 0;$
 $x_{5,10} = 1;$
 $x_{5,11} = 0;$
 $x_{5,12} = 0;$
 $x_{6,1} = 0.03;$
 $x_{6,2} = 0.02;$
 $x_{6,3} = 0.02;$
 $x_{6,4} = 0.02;$
 $x_{6,5} = 0.03;$
 $x_{6,6} = 0;$
 $x_{6,7} = 0;$
 $x_{6,8} = 0;$
 $x_{6,9} = 0;$
 $x_{6,10} = 0;$
 $x_{6,11} = 1;$
 $x_{6,12} = 0;$
 $x_{7,1} = 0.17;$
 $x_{7,2} = 0.13;$
 $x_{7,3} = 0.15;$
 $x_{7,4} = 0.12;$
 $x_{7,5} = 0.02;$
 $x_{7,6} = 0;$
 $x_{7,7} = 0;$
 $x_{7,8} = 0;$
 $x_{7,9} = 0;$
 $x_{7,10} = 0;$
 $x_{7,11} = 0;$
 $x_{7,12} = 1;$

Objective function value

We calculate the value of the objective function by elementwise multiplying column Cb by column P, adding the results of the products.

$$\text{Max}_P = (Cb_1 * P_{01}) + (Cb_{11} * P_2 + (Cb_{21} * P_3 + (Cb_{31} * P_4 + (Cb_{41} * P_5 + (Cb_{51} * P_6 + (Cb_{61} * P_7 = (0 * 1500) + (0 * 1200) + (0 * 300) + (0 * 1000) + (0 * 500) + (0 * 2000) + (0 * 20) = 0;$$

Evaluated Control Variables

We calculate the estimates for each controlled variable, by element-wise multiplying the value from the variable column, by the value from the Cb column, summing up the results of the products, and subtracting the coefficient of the objective function from their sum, with this variable.

$$\text{Max}_{x_1} = ((\text{Cb}_1 * x_{1,1}) + (\text{Cb}_2 * x_{2,1}) + (\text{Cb}_3 * x_{3,1}) + (\text{Cb}_4 * x_{4,1}) + (\text{Cb}_5 * x_{5,1}) + (\text{Cb}_6 * x_{6,1}) + (\text{Cb}_7 * x_{7,1})) - k_{x_1} = ((0 * 0.36) + (0 * 0.22) + (0 * 0.15) + (0 * 0.01) + (0 * 0) + (0 * 0.03) + (0 * 0.17)) - 80 = -80;$$

$$\text{Max}_{x_2} = ((\text{Cb}_1 * x_{1,2}) + (\text{Cb}_2 * x_{2,2}) + (\text{Cb}_3 * x_{3,2}) + (\text{Cb}_4 * x_{4,2}) + (\text{Cb}_5 * x_{5,2}) + (\text{Cb}_6 * x_{6,2}) + (\text{Cb}_7 * x_{7,2})) - k_{x_2} = ((0 * 0.2) + (0 * 0.18) + (0 * 0.06) + (0 * 0.01) + (0 * 0) + (0 * 0.02) + (0 * 0.13)) - 55 = -55;$$

$$\text{Max}_{x_3} = ((\text{Cb}_1 * x_{1,3}) + (\text{Cb}_2 * x_{2,3}) + (\text{Cb}_3 * x_{3,3}) + (\text{Cb}_4 * x_{4,3}) + (\text{Cb}_5 * x_{5,3}) + (\text{Cb}_6 * x_{6,3}) + (\text{Cb}_7 * x_{7,3})) - k_{x_3} = ((0 * 0.25) + (0 * 0.15) + (0 * 0.1) + (0 * 0.01) + (0 * 0) + (0 * 0.02) + (0 * 0.15)) - 40 = -40;$$

$$\text{Max}_{x_4} = ((\text{Cb}_1 * x_{1,4}) + (\text{Cb}_2 * x_{2,4}) + (\text{Cb}_3 * x_{3,4}) + (\text{Cb}_4 * x_{4,4}) + (\text{Cb}_5 * x_{5,4}) + (\text{Cb}_6 * x_{6,4}) + (\text{Cb}_7 * x_{7,4})) - k_{x_4} = ((0 * 0.2) + (0 * 0.15) + (0 * 0.03) + (0 * 0.01) + (0 * 0) + (0 * 0.02) + (0 * 0.12)) - 25 = -25;$$

$$\text{Max}_{x_5} = ((\text{Cb}_1 * x_{1,5}) + (\text{Cb}_2 * x_{2,5}) + (\text{Cb}_3 * x_{3,5}) + (\text{Cb}_4 * x_{4,5}) + (\text{Cb}_5 * x_{5,5}) + (\text{Cb}_6 * x_{6,5}) + (\text{Cb}_7 * x_{7,5})) - k_{x_5} = ((0 * 0.3) + (0 * 0.2) + (0 * 0.15) + (0 * 0.02) + (0 * 0) + (0 * 0.03) + (0 * 0.02)) - 65 = -65;$$

$$\text{Max}_{x_6} = ((\text{Cb}_1 * x_{1,6}) + (\text{Cb}_2 * x_{2,6}) + (\text{Cb}_3 * x_{3,6}) + (\text{Cb}_4 * x_{4,6}) + (\text{Cb}_5 * x_{5,6}) + (\text{Cb}_6 * x_{6,6}) + (\text{Cb}_7 * x_{7,6})) - k_{x_6} = ((0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0)) - 0 = 0;$$

$$\text{Max}_{x_7} = ((\text{Cb}_1 * x_{1,7}) + (\text{Cb}_2 * x_{2,7}) + (\text{Cb}_3 * x_{3,7}) + (\text{Cb}_4 * x_{4,7}) + (\text{Cb}_5 * x_{5,7}) + (\text{Cb}_6 * x_{6,7}) + (\text{Cb}_7 * x_{7,7})) - k_{x_7} = ((0 * 0) + (0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0)) - 0 = 0;$$

$$\text{Max}_{x_8} = ((\text{Cb}_1 * x_{1,8}) + (\text{Cb}_2 * x_{2,8}) + (\text{Cb}_3 * x_{3,8}) + (\text{Cb}_4 * x_{4,8}) + (\text{Cb}_5 * x_{5,8}) + (\text{Cb}_6 * x_{6,8}) + (\text{Cb}_7 * x_{7,8})) - k_{x_8} = ((0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0)) - 0 = 0;$$

$$\text{Max}_{x_9} = ((\text{Cb}_1 * x_{1,9}) + (\text{Cb}_2 * x_{2,9}) + (\text{Cb}_3 * x_{3,9}) + (\text{Cb}_4 * x_{4,9}) + (\text{Cb}_5 * x_{5,9}) + (\text{Cb}_6 * x_{6,9}) + (\text{Cb}_7 * x_{7,9})) - k_{x_9} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (0 * 0) + (0 * 0)) - 0 = 0;$$

$$\text{Max}_{x_{10}} = ((\text{Cb}_1 * x_{1,10}) + (\text{Cb}_2 * x_{2,10}) + (\text{Cb}_3 * x_{3,10}) + (\text{Cb}_4 * x_{4,10}) + (\text{Cb}_5 * x_{5,10}) + (\text{Cb}_6 * x_{6,10}) + (\text{Cb}_7 * x_{7,10})) - k_{x_{10}} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (0 * 0)) - 0 = 0;$$

$$\text{Max}_{x_{11}} = ((\text{Cb}_1 * x_{1,11}) + (\text{Cb}_2 * x_{2,11}) + (\text{Cb}_3 * x_{3,11}) + (\text{Cb}_4 * x_{4,11}) + (\text{Cb}_5 * x_{5,11}) + (\text{Cb}_6 * x_{6,11}) + (\text{Cb}_7 * x_{7,11})) - k_{x_{11}} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (0 * 0)) - 0 = 0;$$

$$\text{Max}_{x_{12}} = ((\text{Cb}_1 * x_{1,12}) + (\text{Cb}_2 * x_{2,12}) + (\text{Cb}_3 * x_{3,12}) + (\text{Cb}_4 * x_{4,12}) + (\text{Cb}_5 * x_{5,12}) + (\text{Cb}_6 * x_{6,12}) + (\text{Cb}_7 * x_{7,12})) - k_{x_{12}} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 1) + (0 * 0)) - 0 = 0;$$

Q column items

Since there are negative values among the estimates of the controlled variables, the current table does not yet have an optimal solution. Therefore, on this basis, we introduce the variable with the smallest negative estimate.

The number of variables in the basis is always constant, so it is necessary to choose which variable to derive from the basis, for which we calculate Q.

The elements of the Q column are calculated by dividing the values from column P by the value from the column corresponding to the variable that is entered in the basis:

$$Q_1 = P_1 / x_{1,1} = 1500 / 0.36 = 4166.67;$$

$$Q_2 = P_2 / x_{2,1} = 1200 / 0.22 = 5454.55;$$

$$Q_3 = P_3 / x_{3,1} = 300 / 0.15 = 2000;$$

$$Q_4 = P_4 / x_{4,1} = 1000 / 0.01 = 83333.33;$$

$$Q_5 = P_5 / x_{5,1} = 500 / 0 = \infty;$$

$$Q_6 = P_6 / x_{6,1} = 2000 / 0.03 = 80000;$$

$$Q_7 = P_7 / x_{7,1} = 20 / 0.17 = 117.65;$$

We deduce from the basis the variable with the least positive value of Q.

At the intersection of the line that corresponds to the variable that is derived from the basis, and the column that corresponds to the variable that is entered into the basis is the resolving element.

This element will allow us to calculate the elements of the table of the next iteration.

Hide

V

V

V

Iteration: 2

B	Cb	P	x ₁	x ₂	x ₃	x ₄	x ₅ ↓	x ₆	x ₇	x ₈	x ₉	x ₁₀	x ₁₁	x ₁₂	Q
			80	55	40	25	65	0	0	0	0	0	0	0	
x ₆	0	1457.65	0	-0.08	-0.07	-0.05	0.26	1	0	0	0	0	0	-2.12	5611.41
x ₇	0	1174.12	0	0.01	-0.04	-0.01	0.18	0	1	0	0	0	0	-1.29	6693.49
x ₈	0	282.35	0	-0.05	-0.03	-0.08	0.13	0	0	1	0	0	0	-0.88	2119.21
x ₉	0	998.59	0	0	-0.01	0	0.01	0	0	0	1	0	0	-0.07	73109.39
x ₁₀	0	499.94	0	0	0	0	0	0	0	0	0	1	0	0	529532.71
x ₁₁	0	1997.06	0	0	-0.01	0	0.02	0	0	0	0	0	1	-0.15	89933.77
x ₁ ←	80	117.65	1	0.76	0.88	0.71	0.11	0	0	0	0	0	0	5.88	1052.63
max		9411.76	0	6.18	30.59	31.47	-56.06	0	0	0	0	0	0	470.59	

Calculation of table elements:

V
V
V

Elements of the column basis (B)

For the results of the calculations of the previous iteration, we remove the variable from the basis x₁₂ and put in her place x₁. All other cells remain unchanged.

Cb column items

Each cell of this column is equal to the coefficient, which corresponds to the base variable in the corresponding row.

- Cb₁ = 0;
- Cb₂ = 0;
- Cb₃ = 0;
- Cb₄ = 0;
- Cb₅ = 0;
- Cb₆ = 0;
- Cb₇ = 80;

Values of variable variables and column P

(The data from the previous iteration is taken as the initial data)

Fill all cells with zeros corresponding to the variable that has just been entered into the basis:

(The resolution element remains unchanged)

- x_{1,1} = 0;
- x_{2,1} = 0;
- x_{3,1} = 0;
- x_{4,1} = 0;

$$x_{5,1} = 0;$$

$$x_{6,1} = 0;$$

We transfer the row with the resolving element from the previous table into the current table, elementwise dividing its values into the resolving element:

$$P_7 = P_7 / x_{7,1} = 20 / 0.17 = 117.65;$$

$$x_{7,1} = x_{7,1} / x_{7,1} = 0.17 / 0.17 = 1;$$

$$x_{7,2} = x_{7,2} / x_{7,1} = 0.13 / 0.17 = 0.76;$$

$$x_{7,3} = x_{7,3} / x_{7,1} = 0.15 / 0.17 = 0.88;$$

$$x_{7,4} = x_{7,4} / x_{7,1} = 0.12 / 0.17 = 0.71;$$

$$x_{7,5} = x_{7,5} / x_{7,1} = 0.02 / 0.17 = 0.11;$$

$$x_{7,6} = x_{7,6} / x_{7,1} = 0 / 0.17 = 0;$$

$$x_{7,7} = x_{7,7} / x_{7,1} = 0 / 0.17 = 0;$$

$$x_{7,8} = x_{7,8} / x_{7,1} = 0 / 0.17 = 0;$$

$$x_{7,9} = x_{7,9} / x_{7,1} = 0 / 0.17 = 0;$$

$$x_{7,10} = x_{7,10} / x_{7,1} = 0 / 0.17 = 0;$$

$$x_{7,11} = x_{7,11} / x_{7,1} = 0 / 0.17 = 0;$$

$$x_{7,12} = x_{7,12} / x_{7,1} = 1 / 0.17 = 5.88;$$

The remaining empty cells, except for the row of estimates and the column Q, are calculated using the rectangle method, relative to the resolving element:

$$P_1 = (P_1 * x_{7,1}) - (x_{1,1} * P_7) / x_{7,1} = ((1500 * 0.17) - (0.36 * 20)) / 0.17 = 1457.65;$$

$$P_2 = (P_2 * x_{7,1}) - (x_{2,1} * P_7) / x_{7,1} = ((1200 * 0.17) - (0.22 * 20)) / 0.17 = 1174.12;$$

$$P_3 = (P_3 * x_{7,1}) - (x_{3,1} * P_7) / x_{7,1} = ((300 * 0.17) - (0.15 * 20)) / 0.17 = 282.35;$$

$$P_4 = (P_4 * x_{7,1}) - (x_{4,1} * P_7) / x_{7,1} = ((1000 * 0.17) - (0.01 * 20)) / 0.17 = 998.59;$$

$$P_5 = (P_5 * x_{7,1}) - (x_{5,1} * P_7) / x_{7,1} = ((500 * 0.17) - (0 * 20)) / 0.17 = 499.94;$$

$$P_6 = (P_6 * x_{7,1}) - (x_{6,1} * P_7) / x_{7,1} = ((2000 * 0.17) - (0.03 * 20)) / 0.17 = 1997.06;$$

$$x_{1,1} = ((x_{1,1} * x_{7,1}) - (x_{1,1} * x_{7,1})) / x_{7,1} = ((0.36 * 0.17) - (0.36 * 0.17)) / 0.17 = 0;$$

$$x_{1,3} = ((x_{1,3} * x_{7,1}) - (x_{1,1} * x_{7,3})) / x_{7,1} = ((0.25 * 0.17) - (0.36 * 0.15)) / 0.17 = -0.07;$$

$$x_{1,4} = ((x_{1,4} * x_{7,1}) - (x_{1,1} * x_{7,4})) / x_{7,1} = ((0.2 * 0.17) - (0.36 * 0.12)) / 0.17 = -0.05;$$

$$x_{1,5} = ((x_{1,5} * x_{7,1}) - (x_{1,1} * x_{7,5})) / x_{7,1} = ((0.3 * 0.17) - (0.36 * 0.02)) / 0.17 = 0.26;$$

$$x_{1,6} = ((x_{1,6} * x_{7,1}) - (x_{1,1} * x_{7,6})) / x_{7,1} = ((1 * 0.17) - (0.36 * 0)) / 0.17 = 1;$$

$$x_{1,7} = ((x_{1,7} * x_{7,1}) - (x_{1,1} * x_{7,7})) / x_{7,1} = ((0 * 0.17) - (0.36 * 0)) / 0.17 = 0;$$

$$x_{1,8} = ((x_{1,8} * x_{7,1}) - (x_{1,1} * x_{7,8})) / x_{7,1} = ((0 * 0.17) - (0.36 * 0)) / 0.17 = 0;$$

$$x_{1,9} = ((x_{1,9} * x_{7,1}) - (x_{1,1} * x_{7,9})) / x_{7,1} = ((0 * 0.17) - (0.36 * 0)) / 0.17 = 0;$$

$$x_{1,10} = ((x_{1,10} * x_{7,1}) - (x_{1,1} * x_{7,10})) / x_{7,1} = ((0 * 0.17) - (0.36 * 0)) / 0.17 = 0;$$

$$x_{1,11} = ((x_{1,11} * x_{7,1}) - (x_{1,1} * x_{7,11})) / x_{7,1} = ((0 * 0.17) - (0.36 * 0)) / 0.17 = 0;$$

$$x_{1,12} = ((x_{1,12} * x_{7,1}) - (x_{1,1} * x_{7,12})) / x_{7,1} = ((0 * 0.17) - (0.36 * 1)) / 0.17 = -2.12;$$

$$x_{2,1} = ((x_{2,1} * x_{7,1}) - (x_{2,1} * x_{7,1})) / x_{7,1} = ((0.22 * 0.17) - (0.22 * 0.17)) / 0.17 = 0;$$

$$x_{2,3} = ((x_{2,3} * x_{7,1}) - (x_{2,1} * x_{7,3})) / x_{7,1} = ((0.15 * 0.17) - (0.22 * 0.15)) / 0.17 = -0.04;$$

$$x_{2,4} = ((x_{2,4} * x_{7,1}) - (x_{2,1} * x_{7,4})) / x_{7,1} = ((0.15 * 0.17) - (0.22 * 0.12)) / 0.17 = -0.01;$$

$$x_{2,5} = ((x_{2,5} * x_{7,1}) - (x_{2,1} * x_{7,5})) / x_{7,1} = ((0.2 * 0.17) - (0.22 * 0.02)) / 0.17 = 0.18;$$

$$x_{2,6} = ((x_{2,6} * x_{7,1}) - (x_{2,1} * x_{7,6})) / x_{7,1} = ((0 * 0.17) - (0.22 * 0)) / 0.17 = 0;$$

$$x_{2,7} = ((x_{2,7} * x_{7,1}) - (x_{2,1} * x_{7,7})) / x_{7,1} = ((1 * 0.17) - (0.22 * 0)) / 0.17 = 1;$$

$$x_{2,8} = ((x_{2,8} * x_{7,1}) - (x_{2,1} * x_{7,8})) / x_{7,1} = ((0 * 0.17) - (0.22 * 0)) / 0.17 = 0;$$

$$x_{2,9} = ((x_{2,9} * x_{7,1}) - (x_{2,1} * x_{7,9})) / x_{7,1} = ((0 * 0.17) - (0.22 * 0)) / 0.17 = 0;$$

$$x_{2,10} = ((x_{2,10} * x_{7,1}) - (x_{2,1} * x_{7,10})) / x_{7,1} = ((0 * 0.17) - (0.22 * 0)) / 0.17 = 0;$$

$$x_{2,11} = ((x_{2,11} * x_{7,1}) - (x_{2,1} * x_{7,11})) / x_{7,1} = ((0 * 0.17) - (0.22 * 0)) / 0.17 = 0;$$

$$x_{2,12} = ((x_{2,12} * x_{7,1}) - (x_{2,1} * x_{7,12})) / x_{7,1} = ((0 * 0.17) - (0.22 * 1)) / 0.17 = -1.29;$$

$$x_{3,1} = ((x_{3,1} * x_{7,1}) - (x_{3,1} * x_{7,1})) / x_{7,1} = ((0.15 * 0.17) - (0.15 * 0.17)) / 0.17 = 0;$$

$$x_{3,3} = ((x_{3,3} * x_{7,1}) - (x_{3,1} * x_{7,3})) / x_{7,1} = ((0.1 * 0.17) - (0.15 * 0.15)) / 0.17 = -0.03;$$

$$x_{3,4} = ((x_{3,4} * x_{7,1}) - (x_{3,1} * x_{7,4})) / x_{7,1} = ((0.03 * 0.17) - (0.15 * 0.12)) / 0.17 = -0.08;$$

$$x_{3,5} = ((x_{3,5} * x_{7,1}) - (x_{3,1} * x_{7,5})) / x_{7,1} = ((0.15 * 0.17) - (0.15 * 0.02)) / 0.17 = 0.13;$$

$$x_{3,6} = ((x_{3,6} * x_{7,1}) - (x_{3,1} * x_{7,6})) / x_{7,1} = ((0 * 0.17) - (0.15 * 0)) / 0.17 = 0;$$

$$x_{3,7} = ((x_{3,7} * x_{7,1}) - (x_{3,1} * x_{7,7})) / x_{7,1} = ((0 * 0.17) - (0.15 * 0)) / 0.17 = 0;$$

$$x_{3,8} = ((x_{3,8} * x_{7,1}) - (x_{3,1} * x_{7,8})) / x_{7,1} = ((1 * 0.17) - (0.15 * 0)) / 0.17 = 1;$$

$$x_{3,9} = ((x_{3,9} * x_{7,1}) - (x_{3,1} * x_{7,9})) / x_{7,1} = ((0 * 0.17) - (0.15 * 0)) / 0.17 = 0;$$

$$x_{3,10} = ((x_{3,10} * x_{7,1}) - (x_{3,1} * x_{7,10})) / x_{7,1} = ((0 * 0.17) - (0.15 * 0)) / 0.17 = 0;$$

$$x_{3,11} = ((x_{3,11} * x_{7,1}) - (x_{3,1} * x_{7,11})) / x_{7,1} = ((0 * 0.17) - (0.15 * 0)) / 0.17 = 0;$$

$$x_{3,12} = ((x_{3,12} * x_{7,1}) - (x_{3,1} * x_{7,12})) / x_{7,1} = ((0 * 0.17) - (0.15 * 1)) / 0.17 = -0.88;$$

$$x_{4,1} = ((x_{4,1} * x_{7,1}) - (x_{4,1} * x_{7,1})) / x_{7,1} = ((0.01 * 0.17) - (0.01 * 0.17)) / 0.17 = 0;$$

$$x_{4,3} = ((x_{4,3} * x_{7,1}) - (x_{4,1} * x_{7,3})) / x_{7,1} = ((0.01 * 0.17) - (0.01 * 0.15)) / 0.17 = -0.01;$$

$$x_{4,4} = ((x_{4,4} * x_{7,1}) - (x_{4,1} * x_{7,4})) / x_{7,1} = ((0.01 * 0.17) - (0.01 * 0.12)) / 0.17 = 0;$$

$$x_{4,5} = ((x_{4,5} * x_{7,1}) - (x_{4,1} * x_{7,5})) / x_{7,1} = ((0.02 * 0.17) - (0.01 * 0.02)) / 0.17 = 0.01;$$

$$x_{4,6} = ((x_{4,6} * x_{7,1}) - (x_{4,1} * x_{7,6})) / x_{7,1} = ((0 * 0.17) - (0.01 * 0)) / 0.17 = 0;$$

$$\begin{aligned}
x_{4,7} &= ((x_{4,7} * x_{7,1}) - (x_{4,1} * x_{7,7})) / x_{7,1} = ((0 * 0.17) - (0.01 * 0)) / 0.17 = 0; \\
x_{4,8} &= ((x_{4,8} * x_{7,1}) - (x_{4,1} * x_{7,8})) / x_{7,1} = ((0 * 0.17) - (0.01 * 0)) / 0.17 = 0; \\
x_{4,9} &= ((x_{4,9} * x_{7,1}) - (x_{4,1} * x_{7,9})) / x_{7,1} = ((1 * 0.17) - (0.01 * 0)) / 0.17 = 1; \\
x_{4,10} &= ((x_{4,10} * x_{7,1}) - (x_{4,1} * x_{7,10})) / x_{7,1} = ((0 * 0.17) - (0.01 * 0)) / 0.17 = 0; \\
x_{4,11} &= ((x_{4,11} * x_{7,1}) - (x_{4,1} * x_{7,11})) / x_{7,1} = ((0 * 0.17) - (0.01 * 0)) / 0.17 = 0; \\
x_{4,12} &= ((x_{4,12} * x_{7,1}) - (x_{4,1} * x_{7,12})) / x_{7,1} = ((0 * 0.17) - (0.01 * 1)) / 0.17 = -0.07; \\
x_{5,1} &= ((x_{5,1} * x_{7,1}) - (x_{5,1} * x_{7,1})) / x_{7,1} = ((0 * 0.17) - (0 * 0.17)) / 0.17 = 0; \\
x_{5,3} &= ((x_{5,3} * x_{7,1}) - (x_{5,1} * x_{7,3})) / x_{7,1} = ((0 * 0.17) - (0 * 0.15)) / 0.17 = 0; \\
x_{5,4} &= ((x_{5,4} * x_{7,1}) - (x_{5,1} * x_{7,4})) / x_{7,1} = ((0 * 0.17) - (0 * 0.12)) / 0.17 = 0; \\
x_{5,5} &= ((x_{5,5} * x_{7,1}) - (x_{5,1} * x_{7,5})) / x_{7,1} = ((0 * 0.17) - (0 * 0.02)) / 0.17 = 0; \\
x_{5,6} &= ((x_{5,6} * x_{7,1}) - (x_{5,1} * x_{7,6})) / x_{7,1} = ((0 * 0.17) - (0 * 0)) / 0.17 = 0; \\
x_{5,7} &= ((x_{5,7} * x_{7,1}) - (x_{5,1} * x_{7,7})) / x_{7,1} = ((0 * 0.17) - (0 * 0)) / 0.17 = 0; \\
x_{5,8} &= ((x_{5,8} * x_{7,1}) - (x_{5,1} * x_{7,8})) / x_{7,1} = ((0 * 0.17) - (0 * 0)) / 0.17 = 0; \\
x_{5,9} &= ((x_{5,9} * x_{7,1}) - (x_{5,1} * x_{7,9})) / x_{7,1} = ((0 * 0.17) - (0 * 0)) / 0.17 = 0; \\
x_{5,10} &= ((x_{5,10} * x_{7,1}) - (x_{5,1} * x_{7,10})) / x_{7,1} = ((1 * 0.17) - (0 * 0)) / 0.17 = 1; \\
x_{5,11} &= ((x_{5,11} * x_{7,1}) - (x_{5,1} * x_{7,11})) / x_{7,1} = ((0 * 0.17) - (0 * 0)) / 0.17 = 0; \\
x_{5,12} &= ((x_{5,12} * x_{7,1}) - (x_{5,1} * x_{7,12})) / x_{7,1} = ((0 * 0.17) - (0 * 1)) / 0.17 = 0; \\
x_{6,1} &= ((x_{6,1} * x_{7,1}) - (x_{6,1} * x_{7,1})) / x_{7,1} = ((0.03 * 0.17) - (0.03 * 0.17)) / 0.17 = 0; \\
x_{6,3} &= ((x_{6,3} * x_{7,1}) - (x_{6,1} * x_{7,3})) / x_{7,1} = ((0.02 * 0.17) - (0.03 * 0.15)) / 0.17 = -0.01; \\
x_{6,4} &= ((x_{6,4} * x_{7,1}) - (x_{6,1} * x_{7,4})) / x_{7,1} = ((0.02 * 0.17) - (0.03 * 0.12)) / 0.17 = 0; \\
x_{6,5} &= ((x_{6,5} * x_{7,1}) - (x_{6,1} * x_{7,5})) / x_{7,1} = ((0.03 * 0.17) - (0.03 * 0.02)) / 0.17 = 0.02; \\
x_{6,6} &= ((x_{6,6} * x_{7,1}) - (x_{6,1} * x_{7,6})) / x_{7,1} = ((0 * 0.17) - (0.03 * 0)) / 0.17 = 0; \\
x_{6,7} &= ((x_{6,7} * x_{7,1}) - (x_{6,1} * x_{7,7})) / x_{7,1} = ((0 * 0.17) - (0.03 * 0)) / 0.17 = 0; \\
x_{6,8} &= ((x_{6,8} * x_{7,1}) - (x_{6,1} * x_{7,8})) / x_{7,1} = ((0 * 0.17) - (0.03 * 0)) / 0.17 = 0; \\
x_{6,9} &= ((x_{6,9} * x_{7,1}) - (x_{6,1} * x_{7,9})) / x_{7,1} = ((0 * 0.17) - (0.03 * 0)) / 0.17 = 0; \\
x_{6,10} &= ((x_{6,10} * x_{7,1}) - (x_{6,1} * x_{7,10})) / x_{7,1} = ((0 * 0.17) - (0.03 * 0)) / 0.17 = 0; \\
x_{6,11} &= ((x_{6,11} * x_{7,1}) - (x_{6,1} * x_{7,11})) / x_{7,1} = ((1 * 0.17) - (0.03 * 0)) / 0.17 = 1; \\
x_{6,12} &= ((x_{6,12} * x_{7,1}) - (x_{6,1} * x_{7,12})) / x_{7,1} = ((0 * 0.17) - (0.03 * 1)) / 0.17 = -0.15;
\end{aligned}$$

Objective function value

We calculate the value of the objective function by elementwise multiplying column Cb by column P, adding the results of the products.

$$\text{Max}_P = (Cb_1 * P_{01}) + (Cb_{11} * P_2 + (Cb_{21} * P_3 + (Cb_{31} * P_4 + (Cb_{41} * P_5 + (Cb_{51} * P_6 + (Cb_{61} * P_7 = (0 * 1457.65) + (0 * 1174.12) + (0 * 282.35) + (0 * 998.59) + (0 * 499.94) + (0 * 1997.06) + (80 * 117.65) = 9411.76;$$

Evaluated Control Variables

We calculate the estimates for each controlled variable, by element-wise multiplying the value from the variable column, by the value from the Cb column, summing up the results of the products, and subtracting the coefficient of the objective function from their sum, with this variable.

$$\begin{aligned}
\text{Max}_{x_1} &= ((Cb_1 * x_{1,1}) + (Cb_2 * x_{2,1}) + (Cb_3 * x_{3,1}) + (Cb_4 * x_{4,1}) + (Cb_5 * x_{5,1}) + (Cb_6 * x_{6,1}) + (Cb_7 * x_{7,1})) - k_{x_1} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (80 * 1)) - 80 = 0; \\
\text{Max}_{x_2} &= ((Cb_1 * x_{1,2}) + (Cb_2 * x_{2,2}) + (Cb_3 * x_{3,2}) + (Cb_4 * x_{4,2}) + (Cb_5 * x_{5,2}) + (Cb_6 * x_{6,2}) + (Cb_7 * x_{7,2})) - k_{x_2} = ((0 * -0.08) + (0 * 0.01) + (0 * -0.05) + (0 * 0) + (0 * 0) + (0 * 0) + (80 * 0.76)) - 55 = 6.18; \\
\text{Max}_{x_3} &= ((Cb_1 * x_{1,3}) + (Cb_2 * x_{2,3}) + (Cb_3 * x_{3,3}) + (Cb_4 * x_{4,3}) + (Cb_5 * x_{5,3}) + (Cb_6 * x_{6,3}) + (Cb_7 * x_{7,3})) - k_{x_3} = ((0 * -0.07) + (0 * -0.04) + (0 * -0.03) + (0 * -0.01) + (0 * 0) + (0 * -0.01) + (80 * 0.88)) - 40 = 30.59; \\
\text{Max}_{x_4} &= ((Cb_1 * x_{1,4}) + (Cb_2 * x_{2,4}) + (Cb_3 * x_{3,4}) + (Cb_4 * x_{4,4}) + (Cb_5 * x_{5,4}) + (Cb_6 * x_{6,4}) + (Cb_7 * x_{7,4})) - k_{x_4} = ((0 * -0.05) + (0 * -0.01) + (0 * -0.08) + (0 * 0) + (0 * 0) + (0 * 0) + (80 * 0.71)) - 25 = 31.47; \\
\text{Max}_{x_5} &= ((Cb_1 * x_{1,5}) + (Cb_2 * x_{2,5}) + (Cb_3 * x_{3,5}) + (Cb_4 * x_{4,5}) + (Cb_5 * x_{5,5}) + (Cb_6 * x_{6,5}) + (Cb_7 * x_{7,5})) - k_{x_5} = ((0 * 0.26) + (0 * 0.18) + (0 * 0.13) + (0 * 0.01) + (0 * 0) + (0 * 0.02) + (80 * 0.11)) - 65 = -56.06; \\
\text{Max}_{x_6} &= ((Cb_1 * x_{1,6}) + (Cb_2 * x_{2,6}) + (Cb_3 * x_{3,6}) + (Cb_4 * x_{4,6}) + (Cb_5 * x_{5,6}) + (Cb_6 * x_{6,6}) + (Cb_7 * x_{7,6})) - k_{x_6} = ((0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (80 * 0)) - 0 = 0; \\
\text{Max}_{x_7} &= ((Cb_1 * x_{1,7}) + (Cb_2 * x_{2,7}) + (Cb_3 * x_{3,7}) + (Cb_4 * x_{4,7}) + (Cb_5 * x_{5,7}) + (Cb_6 * x_{6,7}) + (Cb_7 * x_{7,7})) - k_{x_7} = ((0 * 0) + (0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (80 * 0)) - 0 = 0; \\
\text{Max}_{x_8} &= ((Cb_1 * x_{1,8}) + (Cb_2 * x_{2,8}) + (Cb_3 * x_{3,8}) + (Cb_4 * x_{4,8}) + (Cb_5 * x_{5,8}) + (Cb_6 * x_{6,8}) + (Cb_7 * x_{7,8})) - k_{x_8} = ((0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (80 * 0)) - 0 = 0; \\
\text{Max}_{x_9} &= ((Cb_1 * x_{1,9}) + (Cb_2 * x_{2,9}) + (Cb_3 * x_{3,9}) + (Cb_4 * x_{4,9}) + (Cb_5 * x_{5,9}) + (Cb_6 * x_{6,9}) + (Cb_7 * x_{7,9})) - k_{x_9} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (0 * 0) + (80 * 0)) - 0 = 0; \\
\text{Max}_{x_{10}} &= ((Cb_1 * x_{1,10}) + (Cb_2 * x_{2,10}) + (Cb_3 * x_{3,10}) + (Cb_4 * x_{4,10}) + (Cb_5 * x_{5,10}) + (Cb_6 * x_{6,10}) + (Cb_7 * x_{7,10})) - k_{x_{10}} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (80 * 0)) - 0 = 0; \\
\text{Max}_{x_{11}} &= ((Cb_1 * x_{1,11}) + (Cb_2 * x_{2,11}) + (Cb_3 * x_{3,11}) + (Cb_4 * x_{4,11}) + (Cb_5 * x_{5,11}) + (Cb_6 * x_{6,11}) + (Cb_7 * x_{7,11})) - k_{x_{11}} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (80 * 0)) - 0 = 0;
\end{aligned}$$

$$\text{Max}_{x_{12}} = ((\text{Cb}_1 * x_{1,12}) + (\text{Cb}_2 * x_{2,12}) + (\text{Cb}_3 * x_{3,12}) + (\text{Cb}_4 * x_{4,12}) + (\text{Cb}_5 * x_{5,12}) + (\text{Cb}_6 * x_{6,12}) + (\text{Cb}_7 * x_{7,12})) - k_{x_{12}} = ((0 * - 2.12) + (0 * -1.29) + (0 * -0.88) + (0 * -0.07) + (0 * 0) + (0 * -0.15) + (80 * 5.88)) - 0 = 470.59;$$

Q column items

Since there are negative values among the estimates of the controlled variables, the current table does not yet have an optimal solution. Therefore, on the basis, we introduce the variable with the smallest negative estimate.

The number of variables in the basis is always constant, so it is necessary to choose which variable to derive from the basis, for which we calculate Q.

The elements of the Q column are calculated by dividing the values from column P by the value from the column corresponding to the variable that is entered in the basis:

$$\begin{aligned} Q_1 &= P_1 / x_{1,5} = 1457.65 / 0.26 = 5611.41; \\ Q_2 &= P_2 / x_{2,5} = 1174.12 / 0.18 = 6693.49; \\ Q_3 &= P_3 / x_{3,5} = 282.35 / 0.13 = 2119.21; \\ Q_4 &= P_4 / x_{4,5} = 998.59 / 0.01 = 73109.39; \\ Q_5 &= P_5 / x_{5,5} = 499.94 / 0 = 529532.71; \\ Q_6 &= P_6 / x_{6,5} = 1997.06 / 0.02 = 89933.77; \\ Q_7 &= P_7 / x_{7,5} = 117.65 / 0.11 = 1052.63; \end{aligned}$$

We deduce from the basis the variable with the least positive value of Q.

At the intersection of the line that corresponds to the variable that is derived from the basis, and the column that corresponds to the variable that is entered into the basis is the resolving element.

This element will allow us to calculate the elements of the table of the next iteration.

Hide

V
V
V

Iteration: 3

B	Cb	P	x ₁	x ₂	x ₃	x ₄	x ₅	x ₆	x ₇	x ₈	x ₉	x ₁₀	x ₁₁	x ₁₂	Q
			80	55	40	25	65	0	0	0	0	0	0	0	
x ₆	0	1184.21	-2.32	-1.85	-2.12	-1.69	0	1	0	0	0	0	0	-15.79	
x ₇	0	989.47	-1.57	-1.19	-1.43	-1.11	0	0	1	0	0	0	0	-10.53	
x ₈	0	142.11	-1.19	-0.97	-1.08	-0.92	0	0	0	1	0	0	0	-7.89	
x ₉	0	984.21	-0.12	-0.09	-0.11	-0.08	0	0	0	0	1	0	0	-0.79	
x ₁₀	0	498.95	-0.01	0	-0.01	-0.01	0	0	0	0	0	1	0	-0.05	
x ₁₁	0	1973.68	-0.2	-0.15	-0.18	-0.14	0	0	0	0	0	0	1	-1.32	
x ₅	65	1052.63	8.95	6.84	7.89	6.32	1	0	0	0	0	0	0	52.63	
max		68421.05	501.58	389.74	473.16	385.53	0	0	0	0	0	0	0	3421.05	

Calculation of table elements:

V
V
V

Elements of the column basis (B)

For the results of the calculations of the previous iteration, we remove the variable from the basis x_1 and put in her place x_5 . All other cells remain unchanged.

Cb column items

Each cell of this column is equal to the coefficient, which corresponds to the base variable in the corresponding row.

$Cb_1 = 0;$
 $Cb_2 = 0;$
 $Cb_3 = 0;$
 $Cb_4 = 0;$
 $Cb_5 = 0;$
 $Cb_6 = 0;$
 $Cb_7 = 65;$

Values of variable variables and column P

(The data from the previous iteration is taken as the initial data)

Fill all cells with zeros corresponding to the variable that has just been entered into the basis:
(The resolution element remains unchanged)

$x_{1,5} = 0;$
 $x_{2,5} = 0;$
 $x_{3,5} = 0;$
 $x_{4,5} = 0;$
 $x_{5,5} = 0;$
 $x_{6,5} = 0;$

We transfer the row with the resolving element from the previous table into the current table, elementwise dividing its values into the resolving element:

$P_7 = P_7 / x_{7,5} = 117.65 / 0.11 = 1052.63;$
 $x_{7,1} = x_{7,1} / x_{7,5} = 1 / 0.11 = 8.95;$
 $x_{7,2} = x_{7,2} / x_{7,5} = 0.76 / 0.11 = 6.84;$
 $x_{7,3} = x_{7,3} / x_{7,5} = 0.88 / 0.11 = 7.89;$
 $x_{7,4} = x_{7,4} / x_{7,5} = 0.71 / 0.11 = 6.32;$
 $x_{7,5} = x_{7,5} / x_{7,5} = 0.11 / 0.11 = 1;$
 $x_{7,6} = x_{7,6} / x_{7,5} = 0 / 0.11 = 0;$
 $x_{7,7} = x_{7,7} / x_{7,5} = 0 / 0.11 = 0;$
 $x_{7,8} = x_{7,8} / x_{7,5} = 0 / 0.11 = 0;$
 $x_{7,9} = x_{7,9} / x_{7,5} = 0 / 0.11 = 0;$
 $x_{7,10} = x_{7,10} / x_{7,5} = 0 / 0.11 = 0;$
 $x_{7,11} = x_{7,11} / x_{7,5} = 0 / 0.11 = 0;$
 $x_{7,12} = x_{7,12} / x_{7,5} = 5.88 / 0.11 = 52.63;$

The remaining empty cells, except for the row of estimates and the column Q, are calculated using the rectangle method, relative to the resolving element:

$P_1 = (P_1 * x_{7,5}) - (x_{1,5} * P_7) / x_{7,5} = ((1457.65 * 0.11) - (0.26 * 117.65)) / 0.11 = 1184.21;$
 $P_2 = (P_2 * x_{7,5}) - (x_{2,5} * P_7) / x_{7,5} = ((1174.12 * 0.11) - (0.18 * 117.65)) / 0.11 = 989.47;$
 $P_3 = (P_3 * x_{7,5}) - (x_{3,5} * P_7) / x_{7,5} = ((282.35 * 0.11) - (0.13 * 117.65)) / 0.11 = 142.11;$
 $P_4 = (P_4 * x_{7,5}) - (x_{4,5} * P_7) / x_{7,5} = ((998.59 * 0.11) - (0.01 * 117.65)) / 0.11 = 984.21;$
 $P_5 = (P_5 * x_{7,5}) - (x_{5,5} * P_7) / x_{7,5} = ((499.94 * 0.11) - (0 * 117.65)) / 0.11 = 498.95;$
 $P_6 = (P_6 * x_{7,5}) - (x_{6,5} * P_7) / x_{7,5} = ((1997.06 * 0.11) - (0.02 * 117.65)) / 0.11 = 1973.68;$
 $x_{1,1} = ((x_{1,1} * x_{7,5}) - (x_{1,5} * x_{7,1})) / x_{7,5} = ((0 * 0.11) - (0.26 * 1)) / 0.11 = -2.32;$
 $x_{1,2} = ((x_{1,2} * x_{7,5}) - (x_{1,5} * x_{7,2})) / x_{7,5} = ((-0.08 * 0.11) - (0.26 * 0.76)) / 0.11 = -1.85;$
 $x_{1,3} = ((x_{1,3} * x_{7,5}) - (x_{1,5} * x_{7,3})) / x_{7,5} = ((-0.07 * 0.11) - (0.26 * 0.88)) / 0.11 = -2.12;$
 $x_{1,4} = ((x_{1,4} * x_{7,5}) - (x_{1,5} * x_{7,4})) / x_{7,5} = ((-0.05 * 0.11) - (0.26 * 0.71)) / 0.11 = -1.69;$
 $x_{1,5} = ((x_{1,5} * x_{7,5}) - (x_{1,5} * x_{7,5})) / x_{7,5} = ((0.26 * 0.11) - (0.26 * 0.11)) / 0.11 = 0;$

$$\begin{aligned}
x_{1,7} &= ((x_{1,7} * x_{7,5}) - (x_{1,5} * x_{7,7})) / x_{7,5} = ((0 * 0.11) - (0.26 * 0)) / 0.11 = 0; \\
x_{1,8} &= ((x_{1,8} * x_{7,5}) - (x_{1,5} * x_{7,8})) / x_{7,5} = ((0 * 0.11) - (0.26 * 0)) / 0.11 = 0; \\
x_{1,9} &= ((x_{1,9} * x_{7,5}) - (x_{1,5} * x_{7,9})) / x_{7,5} = ((0 * 0.11) - (0.26 * 0)) / 0.11 = 0; \\
x_{1,10} &= ((x_{1,10} * x_{7,5}) - (x_{1,5} * x_{7,10})) / x_{7,5} = ((0 * 0.11) - (0.26 * 0)) / 0.11 = 0; \\
x_{1,11} &= ((x_{1,11} * x_{7,5}) - (x_{1,5} * x_{7,11})) / x_{7,5} = ((0 * 0.11) - (0.26 * 0)) / 0.11 = 0; \\
x_{1,12} &= ((x_{1,12} * x_{7,5}) - (x_{1,5} * x_{7,12})) / x_{7,5} = ((-2.12 * 0.11) - (0.26 * 5.88)) / 0.11 = -15.79; \\
x_{2,1} &= ((x_{2,1} * x_{7,5}) - (x_{2,5} * x_{7,1})) / x_{7,5} = ((0 * 0.11) - (0.18 * 1)) / 0.11 = -1.57; \\
x_{2,2} &= ((x_{2,2} * x_{7,5}) - (x_{2,5} * x_{7,2})) / x_{7,5} = ((0.01 * 0.11) - (0.18 * 0.76)) / 0.11 = -1.19; \\
x_{2,3} &= ((x_{2,3} * x_{7,5}) - (x_{2,5} * x_{7,3})) / x_{7,5} = ((-0.04 * 0.11) - (0.18 * 0.88)) / 0.11 = -1.43; \\
x_{2,4} &= ((x_{2,4} * x_{7,5}) - (x_{2,5} * x_{7,4})) / x_{7,5} = ((-0.01 * 0.11) - (0.18 * 0.71)) / 0.11 = -1.11; \\
x_{2,5} &= ((x_{2,5} * x_{7,5}) - (x_{2,5} * x_{7,5})) / x_{7,5} = ((0.18 * 0.11) - (0.18 * 0.11)) / 0.11 = 0; \\
x_{2,7} &= ((x_{2,7} * x_{7,5}) - (x_{2,5} * x_{7,7})) / x_{7,5} = ((1 * 0.11) - (0.18 * 0)) / 0.11 = 1; \\
x_{2,8} &= ((x_{2,8} * x_{7,5}) - (x_{2,5} * x_{7,8})) / x_{7,5} = ((0 * 0.11) - (0.18 * 0)) / 0.11 = 0; \\
x_{2,9} &= ((x_{2,9} * x_{7,5}) - (x_{2,5} * x_{7,9})) / x_{7,5} = ((0 * 0.11) - (0.18 * 0)) / 0.11 = 0; \\
x_{2,10} &= ((x_{2,10} * x_{7,5}) - (x_{2,5} * x_{7,10})) / x_{7,5} = ((0 * 0.11) - (0.18 * 0)) / 0.11 = 0; \\
x_{2,11} &= ((x_{2,11} * x_{7,5}) - (x_{2,5} * x_{7,11})) / x_{7,5} = ((0 * 0.11) - (0.18 * 0)) / 0.11 = 0; \\
x_{2,12} &= ((x_{2,12} * x_{7,5}) - (x_{2,5} * x_{7,12})) / x_{7,5} = ((-1.29 * 0.11) - (0.18 * 5.88)) / 0.11 = -10.53; \\
x_{3,1} &= ((x_{3,1} * x_{7,5}) - (x_{3,5} * x_{7,1})) / x_{7,5} = ((0 * 0.11) - (0.13 * 1)) / 0.11 = -1.19; \\
x_{3,2} &= ((x_{3,2} * x_{7,5}) - (x_{3,5} * x_{7,2})) / x_{7,5} = ((-0.05 * 0.11) - (0.13 * 0.76)) / 0.11 = -0.97; \\
x_{3,3} &= ((x_{3,3} * x_{7,5}) - (x_{3,5} * x_{7,3})) / x_{7,5} = ((-0.03 * 0.11) - (0.13 * 0.88)) / 0.11 = -1.08; \\
x_{3,4} &= ((x_{3,4} * x_{7,5}) - (x_{3,5} * x_{7,4})) / x_{7,5} = ((-0.08 * 0.11) - (0.13 * 0.71)) / 0.11 = -0.92; \\
x_{3,5} &= ((x_{3,5} * x_{7,5}) - (x_{3,5} * x_{7,5})) / x_{7,5} = ((0.13 * 0.11) - (0.13 * 0.11)) / 0.11 = 0; \\
x_{3,7} &= ((x_{3,7} * x_{7,5}) - (x_{3,5} * x_{7,7})) / x_{7,5} = ((0 * 0.11) - (0.13 * 0)) / 0.11 = 0; \\
x_{3,8} &= ((x_{3,8} * x_{7,5}) - (x_{3,5} * x_{7,8})) / x_{7,5} = ((1 * 0.11) - (0.13 * 0)) / 0.11 = 1; \\
x_{3,9} &= ((x_{3,9} * x_{7,5}) - (x_{3,5} * x_{7,9})) / x_{7,5} = ((0 * 0.11) - (0.13 * 0)) / 0.11 = 0; \\
x_{3,10} &= ((x_{3,10} * x_{7,5}) - (x_{3,5} * x_{7,10})) / x_{7,5} = ((0 * 0.11) - (0.13 * 0)) / 0.11 = 0; \\
x_{3,11} &= ((x_{3,11} * x_{7,5}) - (x_{3,5} * x_{7,11})) / x_{7,5} = ((0 * 0.11) - (0.13 * 0)) / 0.11 = 0; \\
x_{3,12} &= ((x_{3,12} * x_{7,5}) - (x_{3,5} * x_{7,12})) / x_{7,5} = ((-0.88 * 0.11) - (0.13 * 5.88)) / 0.11 = -7.89; \\
x_{4,1} &= ((x_{4,1} * x_{7,5}) - (x_{4,5} * x_{7,1})) / x_{7,5} = ((0 * 0.11) - (0.01 * 1)) / 0.11 = -0.12; \\
x_{4,2} &= ((x_{4,2} * x_{7,5}) - (x_{4,5} * x_{7,2})) / x_{7,5} = ((0 * 0.11) - (0.01 * 0.76)) / 0.11 = -0.09; \\
x_{4,3} &= ((x_{4,3} * x_{7,5}) - (x_{4,5} * x_{7,3})) / x_{7,5} = ((-0.01 * 0.11) - (0.01 * 0.88)) / 0.11 = -0.11; \\
x_{4,4} &= ((x_{4,4} * x_{7,5}) - (x_{4,5} * x_{7,4})) / x_{7,5} = ((0 * 0.11) - (0.01 * 0.71)) / 0.11 = -0.08; \\
x_{4,5} &= ((x_{4,5} * x_{7,5}) - (x_{4,5} * x_{7,5})) / x_{7,5} = ((0.01 * 0.11) - (0.01 * 0.11)) / 0.11 = 0; \\
x_{4,7} &= ((x_{4,7} * x_{7,5}) - (x_{4,5} * x_{7,7})) / x_{7,5} = ((0 * 0.11) - (0.01 * 0)) / 0.11 = 0; \\
x_{4,8} &= ((x_{4,8} * x_{7,5}) - (x_{4,5} * x_{7,8})) / x_{7,5} = ((0 * 0.11) - (0.01 * 0)) / 0.11 = 0; \\
x_{4,9} &= ((x_{4,9} * x_{7,5}) - (x_{4,5} * x_{7,9})) / x_{7,5} = ((1 * 0.11) - (0.01 * 0)) / 0.11 = 1; \\
x_{4,10} &= ((x_{4,10} * x_{7,5}) - (x_{4,5} * x_{7,10})) / x_{7,5} = ((0 * 0.11) - (0.01 * 0)) / 0.11 = 0; \\
x_{4,11} &= ((x_{4,11} * x_{7,5}) - (x_{4,5} * x_{7,11})) / x_{7,5} = ((0 * 0.11) - (0.01 * 0)) / 0.11 = 0; \\
x_{4,12} &= ((x_{4,12} * x_{7,5}) - (x_{4,5} * x_{7,12})) / x_{7,5} = ((-0.07 * 0.11) - (0.01 * 5.88)) / 0.11 = -0.79; \\
x_{5,1} &= ((x_{5,1} * x_{7,5}) - (x_{5,5} * x_{7,1})) / x_{7,5} = ((0 * 0.11) - (0 * 1)) / 0.11 = -0.01; \\
x_{5,2} &= ((x_{5,2} * x_{7,5}) - (x_{5,5} * x_{7,2})) / x_{7,5} = ((0 * 0.11) - (0 * 0.76)) / 0.11 = 0; \\
x_{5,3} &= ((x_{5,3} * x_{7,5}) - (x_{5,5} * x_{7,3})) / x_{7,5} = ((0 * 0.11) - (0 * 0.88)) / 0.11 = -0.01; \\
x_{5,4} &= ((x_{5,4} * x_{7,5}) - (x_{5,5} * x_{7,4})) / x_{7,5} = ((0 * 0.11) - (0 * 0.71)) / 0.11 = -0.01; \\
x_{5,5} &= ((x_{5,5} * x_{7,5}) - (x_{5,5} * x_{7,5})) / x_{7,5} = ((0 * 0.11) - (0 * 0.11)) / 0.11 = 0; \\
x_{5,7} &= ((x_{5,7} * x_{7,5}) - (x_{5,5} * x_{7,7})) / x_{7,5} = ((0 * 0.11) - (0 * 0)) / 0.11 = 0; \\
x_{5,8} &= ((x_{5,8} * x_{7,5}) - (x_{5,5} * x_{7,8})) / x_{7,5} = ((0 * 0.11) - (0 * 0)) / 0.11 = 0; \\
x_{5,9} &= ((x_{5,9} * x_{7,5}) - (x_{5,5} * x_{7,9})) / x_{7,5} = ((0 * 0.11) - (0 * 0)) / 0.11 = 0; \\
x_{5,10} &= ((x_{5,10} * x_{7,5}) - (x_{5,5} * x_{7,10})) / x_{7,5} = ((1 * 0.11) - (0 * 0)) / 0.11 = 1; \\
x_{5,11} &= ((x_{5,11} * x_{7,5}) - (x_{5,5} * x_{7,11})) / x_{7,5} = ((0 * 0.11) - (0 * 0)) / 0.11 = 0; \\
x_{5,12} &= ((x_{5,12} * x_{7,5}) - (x_{5,5} * x_{7,12})) / x_{7,5} = ((0 * 0.11) - (0 * 5.88)) / 0.11 = -0.05; \\
x_{6,1} &= ((x_{6,1} * x_{7,5}) - (x_{6,5} * x_{7,1})) / x_{7,5} = ((0 * 0.11) - (0.02 * 1)) / 0.11 = -0.2; \\
x_{6,2} &= ((x_{6,2} * x_{7,5}) - (x_{6,5} * x_{7,2})) / x_{7,5} = ((0 * 0.11) - (0.02 * 0.76)) / 0.11 = -0.15; \\
x_{6,3} &= ((x_{6,3} * x_{7,5}) - (x_{6,5} * x_{7,3})) / x_{7,5} = ((-0.01 * 0.11) - (0.02 * 0.88)) / 0.11 = -0.18; \\
x_{6,4} &= ((x_{6,4} * x_{7,5}) - (x_{6,5} * x_{7,4})) / x_{7,5} = ((0 * 0.11) - (0.02 * 0.71)) / 0.11 = -0.14; \\
x_{6,5} &= ((x_{6,5} * x_{7,5}) - (x_{6,5} * x_{7,5})) / x_{7,5} = ((0.02 * 0.11) - (0.02 * 0.11)) / 0.11 = 0; \\
x_{6,7} &= ((x_{6,7} * x_{7,5}) - (x_{6,5} * x_{7,7})) / x_{7,5} = ((0 * 0.11) - (0.02 * 0)) / 0.11 = 0; \\
x_{6,8} &= ((x_{6,8} * x_{7,5}) - (x_{6,5} * x_{7,8})) / x_{7,5} = ((0 * 0.11) - (0.02 * 0)) / 0.11 = 0; \\
x_{6,9} &= ((x_{6,9} * x_{7,5}) - (x_{6,5} * x_{7,9})) / x_{7,5} = ((0 * 0.11) - (0.02 * 0)) / 0.11 = 0; \\
x_{6,10} &= ((x_{6,10} * x_{7,5}) - (x_{6,5} * x_{7,10})) / x_{7,5} = ((0 * 0.11) - (0.02 * 0)) / 0.11 = 0; \\
x_{6,11} &= ((x_{6,11} * x_{7,5}) - (x_{6,5} * x_{7,11})) / x_{7,5} = ((1 * 0.11) - (0.02 * 0)) / 0.11 = 1; \\
x_{6,12} &= ((x_{6,12} * x_{7,5}) - (x_{6,5} * x_{7,12})) / x_{7,5} = ((-0.15 * 0.11) - (0.02 * 5.88)) / 0.11 = -1.32;
\end{aligned}$$

Objective function value

We calculate the value of the objective function by elementwise multiplying column Cb by column P, adding the results of the products.

$$\text{Max}_P = (\text{Cb}_1 * P_{01}) + (\text{Cb}_{11} * P_2 + (\text{Cb}_{21} * P_3 + (\text{Cb}_{31} * P_4 + (\text{Cb}_{41} * P_5 + (\text{Cb}_{51} * P_6 + (\text{Cb}_{61} * P_7 = (0 * 1184.21) + (0 * 989.47) + (0 * 142.11) + (0 * 984.21) + (0 * 498.95) + (0 * 1973.68) + (65 * 1052.63) = 68421.05;$$

Evaluated Control Variables

We calculate the estimates for each controlled variable, by element-wise multiplying the value from the variable column, by the value from the Cb column, summing up the results of the products, and subtracting the coefficient of the objective function from their sum, with this variable.

$$\text{Max}_{x_1} = ((\text{Cb}_1 * x_{1,1}) + (\text{Cb}_2 * x_{2,1}) + (\text{Cb}_3 * x_{3,1}) + (\text{Cb}_4 * x_{4,1}) + (\text{Cb}_5 * x_{5,1}) + (\text{Cb}_6 * x_{6,1}) + (\text{Cb}_7 * x_{7,1})) - k_{x1} = ((0 * -2.32) + (0 * -1.57) + (0 * -1.19) + (0 * -0.12) + (0 * -0.01) + (0 * -0.2) + (65 * 8.95)) - 80 = 501.58;$$

$$\text{Max}_{x_2} = ((\text{Cb}_1 * x_{1,2}) + (\text{Cb}_2 * x_{2,2}) + (\text{Cb}_3 * x_{3,2}) + (\text{Cb}_4 * x_{4,2}) + (\text{Cb}_5 * x_{5,2}) + (\text{Cb}_6 * x_{6,2}) + (\text{Cb}_7 * x_{7,2})) - k_{x2} = ((0 * -1.85) + (0 * -1.19) + (0 * -0.97) + (0 * -0.09) + (0 * 0) + (0 * -0.15) + (65 * 6.84)) - 55 = 389.74;$$

$$\text{Max}_{x_3} = ((\text{Cb}_1 * x_{1,3}) + (\text{Cb}_2 * x_{2,3}) + (\text{Cb}_3 * x_{3,3}) + (\text{Cb}_4 * x_{4,3}) + (\text{Cb}_5 * x_{5,3}) + (\text{Cb}_6 * x_{6,3}) + (\text{Cb}_7 * x_{7,3})) - k_{x3} = ((0 * -2.12) + (0 * -1.43) + (0 * -1.08) + (0 * -0.11) + (0 * -0.01) + (0 * -0.18) + (65 * 7.89)) - 40 = 473.16;$$

$$\text{Max}_{x_4} = ((\text{Cb}_1 * x_{1,4}) + (\text{Cb}_2 * x_{2,4}) + (\text{Cb}_3 * x_{3,4}) + (\text{Cb}_4 * x_{4,4}) + (\text{Cb}_5 * x_{5,4}) + (\text{Cb}_6 * x_{6,4}) + (\text{Cb}_7 * x_{7,4})) - k_{x4} = ((0 * -1.69) + (0 * -1.11) + (0 * -0.92) + (0 * -0.08) + (0 * -0.01) + (0 * -0.14) + (65 * 6.32)) - 25 = 385.53;$$

$$\text{Max}_{x_5} = ((\text{Cb}_1 * x_{1,5}) + (\text{Cb}_2 * x_{2,5}) + (\text{Cb}_3 * x_{3,5}) + (\text{Cb}_4 * x_{4,5}) + (\text{Cb}_5 * x_{5,5}) + (\text{Cb}_6 * x_{6,5}) + (\text{Cb}_7 * x_{7,5})) - k_{x5} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (65 * 1)) - 65 = 0;$$

$$\text{Max}_{x_6} = ((\text{Cb}_1 * x_{1,6}) + (\text{Cb}_2 * x_{2,6}) + (\text{Cb}_3 * x_{3,6}) + (\text{Cb}_4 * x_{4,6}) + (\text{Cb}_5 * x_{5,6}) + (\text{Cb}_6 * x_{6,6}) + (\text{Cb}_7 * x_{7,6})) - k_{x6} = ((0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (65 * 0)) - 0 = 0;$$

$$\text{Max}_{x_7} = ((\text{Cb}_1 * x_{1,7}) + (\text{Cb}_2 * x_{2,7}) + (\text{Cb}_3 * x_{3,7}) + (\text{Cb}_4 * x_{4,7}) + (\text{Cb}_5 * x_{5,7}) + (\text{Cb}_6 * x_{6,7}) + (\text{Cb}_7 * x_{7,7})) - k_{x7} = ((0 * 0) + (0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (65 * 0)) - 0 = 0;$$

$$\text{Max}_{x_8} = ((\text{Cb}_1 * x_{1,8}) + (\text{Cb}_2 * x_{2,8}) + (\text{Cb}_3 * x_{3,8}) + (\text{Cb}_4 * x_{4,8}) + (\text{Cb}_5 * x_{5,8}) + (\text{Cb}_6 * x_{6,8}) + (\text{Cb}_7 * x_{7,8})) - k_{x8} = ((0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (65 * 0)) - 0 = 0;$$

$$\text{Max}_{x_9} = ((\text{Cb}_1 * x_{1,9}) + (\text{Cb}_2 * x_{2,9}) + (\text{Cb}_3 * x_{3,9}) + (\text{Cb}_4 * x_{4,9}) + (\text{Cb}_5 * x_{5,9}) + (\text{Cb}_6 * x_{6,9}) + (\text{Cb}_7 * x_{7,9})) - k_{x9} = ((0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (0 * 0) + (0 * 0) + (65 * 0)) - 0 = 0;$$

$$\text{Max}_{x_{10}} = ((\text{Cb}_1 * x_{1,10}) + (\text{Cb}_2 * x_{2,10}) + (\text{Cb}_3 * x_{3,10}) + (\text{Cb}_4 * x_{4,10}) + (\text{Cb}_5 * x_{5,10}) + (\text{Cb}_6 * x_{6,10}) + (\text{Cb}_7 * x_{7,10})) - k_{x_{10}} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 1) + (0 * 0) + (65 * 0)) - 0 = 0;$$

$$\text{Max}_{x_{11}} = ((\text{Cb}_1 * x_{1,11}) + (\text{Cb}_2 * x_{2,11}) + (\text{Cb}_3 * x_{3,11}) + (\text{Cb}_4 * x_{4,11}) + (\text{Cb}_5 * x_{5,11}) + (\text{Cb}_6 * x_{6,11}) + (\text{Cb}_7 * x_{7,11})) - k_{x_{11}} = ((0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 0) + (0 * 1) + (65 * 0)) - 0 = 0;$$

$$\text{Max}_{x_{12}} = ((\text{Cb}_1 * x_{1,12}) + (\text{Cb}_2 * x_{2,12}) + (\text{Cb}_3 * x_{3,12}) + (\text{Cb}_4 * x_{4,12}) + (\text{Cb}_5 * x_{5,12}) + (\text{Cb}_6 * x_{6,12}) + (\text{Cb}_7 * x_{7,12})) - k_{x_{12}} = ((0 * -15.79) + (0 * -10.53) + (0 * -7.89) + (0 * -0.79) + (0 * -0.05) + (0 * -1.32) + (65 * 52.63)) - 0 = 3421.05;$$

Answer:

Since there are no negative values among the estimates of the controlled variables, the current table has an optimal solution.

The value of the objective function:

$$F^* = 68421.05;$$

The variables that are present in the basis are equal to the corresponding cells of the column P, all other variables are equal to zero:

- $x_1 = 0;$
- $x_2 = 0;$
- $x_3 = 0;$
- $x_4 = 0;$
- $x_5 = 1052.63;$

Hide
V
V
V

Answer:

$$F^* = 68421.05$$

$$X^* = (0; 0; 0; 0; 1052.63)$$
